

NEXT-GENERATION MATH ACCUPLACER TEST REVIEW BOOKLET

Next Generation

Quantitative Reasoning Algebra and Statistics

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Each subsection in this practice book is followed by practice problems and answers. Make sure to check your answers!

This practice book contains a lot of material. The intention of including so much information is to make sure you're prepared for the test. You can start at the very beginning and work your way up, or you can focus on specific sections you need help with. Make sure you ask a tutor for help if you aren't sure about something!

About

Here is some information about the MSU Tutoring Center and the Placement Test.

MSU Denver Tutoring Center Mission

The mission of the MSU Denver Tutoring Center is to increase student persistence by offering free student-centered academic support across disciplines. The Tutoring Center is not only for students who are having difficulty with course material, but also for students who are looking to excel. Whether your goal is to catch up, keep up, or do better in your studies, the Tutoring Center is here to help you by offering free one-on-one and group academic support in a variety of subjects. We promote an environment that is welcoming to the diverse student body of MSU Denver by providing professionally trained tutors who are competent in subject material and areas such as diversity, learning styles, and communication.

What is the Next-Generation Accuplacer?

The Next-Generation Quantitative Reasoning, Algebra, and Statistics (QAS) placement test is a computer adaptive assessment of test-takers' ability for selected mathematics content. Questions will focus on a range of topics including computing with rational numbers, applying ratios and proportional reasoning, creating linear expressions and equations, graphing and applying linear equations, understanding probability and set notation, and interpreting graphical displays. In addition, questions may assess a student's math ability via computational or fluency skills, conceptual understanding, or the capacity to apply mathematics presented in a context. All questions are multiple choice in format and appear discretely (standalone) across the assessment. The following knowledge and skill categories are assessed:

- Rational numbers
- Ratio and proportional relationships
- Exponents
- Algebraic expressions
- Linear equations
- Linear applications
- Probability and sets
- Descriptive statistics
- Geometry concepts

Test Taking Tips

- Take your time on the test. Students tend to get higher scores the longer they spend finding and double checking their answers.
- <u>Read carefully</u>! Pay careful attention to how questions are worded and what they want you to find.
- Make use of the scratch paper provided- you are going to make mistakes if you try to solve problems in your head.
- The test is multiple choice, so use that to your advantage. If you can't remember how to approach a problem, maybe you'll find the correct answer by plugging the multiple choice answers into the original question!
- If you don't remember some of the rules for exponents, radicals, or other operations, try constructing a simpler example that you know better and solve that one. The rules may become clear to you once you try the simple example.
- Another strategy for remembering the many rules involved with operations is to break things down to their smaller pieces. Under pressure, it's easier to see what's going on with $x \cdot x \cdot x \cdot x$ instead of x^4 .
- The test uses an algorithm called *branching*. This means that it changes its difficulty level each time you answer a question, depending on how well you've done. If you are getting questions wrong, the test will make itself easier. If you are getting questions correct, the test will make itself harder. It's a good sign if the test seems to be getting more challenging. (You earn more points for more difficult questions).
- Eat before you take the test. This will help you concentrate better.
- Get good sleep the night before you take the test. If you don't sleep well the night before, postpone it.
- Don't be intimidated by the questions or the test in general. It's okay to have anxiety- recognize it and do your best to relax so you can concentrate.
- Review for this test right before you go to sleep at night.



Rational Numbers

1.1 Useful Memorization

A calculator will <u>not</u> be allowed on the test, so make sure you have some of this memorized or know how to calculate it!

Multiplication Table

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Perfect Squares

$1^2 = 1$	$4^2 = 16$	$7^2 = 49$	$10^2 = 100$
$2^2 = 4$	$5^2 = 25$	$8^2 = 64$	$11^2 = 121$
$3^2 = 9$	$6^2 = 36$	$9^2 = 81$	$12^2 = 144$

1.2 Absolute Value

The absolute value | | functions as a grouping and an operation. It groups things together like parentheses, and it also changes negative numbers into positive numbers.

One way to think about the absolute value function is a measurement of distance. Consider |3| and |-3|. How far away is 3 from zero on a number line? How far away is -3 from zero on a number line?



Since the number 3 is three units away from zero, |3| = 3. Since the number -3 is three units away from zero, |-3| = 3.

Example

An easy way to think about absolute value is that it turns negative numbers positive, and keeps positive numbers positive. Once the absolute value has been evaluated (turned positive), do not continue to write the vertical bars | |.

$$|-2| = 2$$
 $|2| = 2$

Practice Problems

Evaluate the following.

1) -1	6)	-400
--------	----	------

- 2) |1| 7) |0|
- 3) |3| 8) |3-2|
- 4) |-7| 9) |2-3|
- 5) |-10| 10) $|1 \div 2|$

Answers

1)	1	-1 is one unit away from zero.
2)	1	1 is one unit away from zero.
3)	3	3 is three units away from zero.
4)	7	-7 is seven units away from zero.
5)	10	-10 is ten units away from zero.
6)	400	-400 is 400 units away from zero.
7)	0	0 is zero units away from itself.
8)	1	3-2 = 1 , and 1 is one unit away from zero.
9)	1	2-3 = -1 , and -1 is one unit away from zero.
10)	$\frac{1}{2}$	$ 1 \div 2 = \left \frac{1}{2}\right $, and $\frac{1}{2}$ is a half unit away from zero.

1.3 Order of Operations

The different operations in mathematical expressions (addition, subtraction, multiplication, division, exponentiation, and grouping) must be evaluated in a specific order. This structure gives us consistent answers when we simplify expressions.

Why is it needed?

Consider the expression $4 \times 2 + 1$. If multiplication is evaluated before addition, $4 \times 2 + 1$ becomes 8 + 1 which then becomes 9. If addition is evaluated before multiplication, $4 \times 2 + 1$ becomes 4×3 which then becomes 12. Since $9 \neq 12$, evaluating in any order we want doesn't give us consistent answers.

The Order of Operations

1) Parentheses (evaluate the inside)	()[]{}
2) Exponents	$x^2 \leftarrow This number$
3) Multiplication and Division (from left to right)	×÷
4) Addition and Subtraction (from left to right)	+ -

Notice that Multiplication and Division share the third spot and Addition and Subtraction share the fourth spot. In these cases, evaluate from left to right (see examples below).

PEMDAS is the mnemonic used to remember the Order of Operations. Start with Parentheses and end with addition and subtraction. (Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction).

Starting point:	$4 \times 2 + 1$
Multiplication happens before addition:	8 + 1
All that's left is to add 8 and 1:	9

Example

Starting point:	$4 \times (2 + 1)$
Parentheses are first (evaluate the inside):	4×3
All that's left is to multiply 4 and 3:	12

Example

Starting point:	$4 + 3 \times 2 - 1$
Multiplication happens before addition or subtraction:	4 + 6 - 1
Addition and subtraction happens from left to right:	10 - 1
	9

Note about exponents

Pay close attention to notation with exponents! The position of a negative sign can change an entire problem.

$$-2^{2} = -(2)(2) = -4$$

(-2)² = (-2)(-2) = 4

In the first expression, the negative is sitting outside of the exponent operation. In the second expression, it is included and cancels out.

Starting point:	$1 + (5 \times 3 - 5)^2$
Start with the operations in parentheses:	$1 + (5 \times 3 - 5)^2$
Within the parentheses, multiply first:	$1 + (15 - 5)^2$
Within the parentheses, subtract second:	$1 + (10)^2$
Now the exponent is evaluated:	1 + 100
Finally, add:	101

Example

Starting point:	$8 \div 4 \times 5 \div 2$
Multiplication and division are equal in	
PEMDAS, so proceed from left to right:	$2 \times 5 \div 2$
	$10 \div 2$
	5

Example

Starting point:

Addition and subtraction are equal in PEMDAS, so proceed from left to right:

$$3 - 2 - 1 + 2$$

$$1 - 1 + 2$$

$$0 + 2$$

$$2$$

Starting point:	$\frac{2 \times 2 + 3}{7^2}$
Fractions are the same as division, so rewrite:	$(2 \times 2 + 3) \div (7^2)$
Start with the operations in the parentheses:	$(7) \div (49)$
Then divide 7 and 49:	$\frac{7}{49}$
Reduce	$\frac{1}{7}$

Note about absolute value

The absolute value parentheses || function in two ways. They group an expression just like any other parentheses, but they also turn whatever simplified number they contain into the positive version of that number.

|-3| = 3 and |3| = 3

 $|3-8| \times (\sqrt{100} - 5)$

 $|-5| \times (10 - 5)$ $|-5| \times 5$

 5×5

25

Example

Starting point:

Start with the operations in the parentheses:

The absolutely turns the -5 positive now:

Perform the multiplication:

Starting point:

With nested parentheses, start with the innermost ones:

Perform the multiplication:

$$\begin{bmatrix} 3 \times (4 - 3) \end{bmatrix}$$
$$\begin{bmatrix} 3 \times 1 \end{bmatrix}$$
$$\begin{bmatrix} 3 \times 1 \end{bmatrix}$$

Note about radicals

Radicals are technically exponents (see section 3.3). So radicals will be evaluated at the same time as exponents:

$$\sqrt{11+5}+2$$

$$\sqrt{16}+2$$

$$4+2$$

$$6$$

Practice Problems

Evaluate the following.

1) $(5+3) \cdot 2$	6) $5 + (3 \cdot 6 + 2)$	11) $\frac{(5\cdot 2)(6-3)}{6\div 2}$
2) 5 + (3 · 2)	7) $-2^2 + 3^2$	12) $\frac{(4+2)^1+2^3}{ 2 }$
3) $2 \cdot (8+5) - 2$	8) $(-2)^2 + 3^2$	13) $0 \cdot (3 + 6 - 8)^2 - 14$
4) $2 \cdot 8 + 5 - 2$	9) $\frac{(-2)^{2} \cdot 2}{2^{3}}$	14) $\left \frac{10^2}{-100-50} \right $
5) $(2+3)^2 - 2-4 $	10) $((5-3)^2)^2$	15) $(6 + [2 - 2^3]) + 2^0$

Answers

1)	16	$(5+3) \cdot 2 = 8 \cdot 2 = 16$
2)	11	$5 + (3 \cdot 2) = 5 + 6 = 11$
3)	24	$2 \cdot (8+5) - 2 = 2 \cdot 13 - 2 = 26 - 2 = 24$
4)	19	$2 \cdot 8 + 5 - 2 = 16 + 5 - 2 = 21 - 2 = 19$
5)	23	$(2+3)^2 - 2-4 = (5)^2 - -2 = 25 - 2 = 23$
6)	25	$5 + (3 \cdot 6 + 2) = 5 + (18 + 2) = 5 + (20) = 25$
7)	5	$-2^2 + 3^2 = -4 + 9 = 5$
8)	13	$(-2)^2 + 3^2 = 4 + 9 = 13$
9)	1	$\frac{(-2)^2 \cdot 2}{2^3} = \frac{4 \cdot 2}{8} = \frac{8}{8} = 1$
10)	16	$((5-3)^2)^2 = ((2)^2)^2 = (4)^2 = 16$
11)	10	$\frac{(5\cdot 2)(6-3)}{6\div 2} = \frac{(10)(3)}{3} = \frac{30}{3} = 10$
12)	7	$\frac{(4+2)^1+2^3}{ 2 } = \frac{(6)^1+8}{2} = \frac{6+8}{2} = \frac{14}{2} = 7$
13)	-14	$0 \cdot (3+6-8)^2 - 14 = 0 \cdot (9-8)^2 - 14 = 0 \cdot (1)^2 - 14 = 0 \cdot 1 - 14 = 0 - 14 = -14$
14)	$\frac{2}{3}$	$\left \frac{10^2}{-100-50}\right = \left \frac{100}{-150}\right = \left \frac{2}{-3}\right = \frac{2}{3}$
15)	1	$(6 + [2 - 2^3]) + 2^0 = (6 + [2 - 8]) + 2^0 = (6 - 6) + 2^0 = 0 + 2^0 = 1$

1.4 Substitution

Simply explained, substitution is when you replace the letters in an expression with numbers.

The Process

Replace the variables in the original expression with their assigned values.

Example			
Evaluate x^2y where $x = 2$ and $y = 3$.			
Starting point:	x^2y		
Replace x with 2 and y with 3	$(2)^2 \cdot (3)$		
Evaluate the exponent	4 · 3		
Multiply	12		

Example

Evaluate x^2y where x = -2 and y = -3.

Starting point:	x^2y
Replace x with -2 and y with -3	$(-2)^2 \cdot (-3)$
Evaluate the exponent	$4 \cdot -3$
Multiply	-12

Note

It is important to plug in negative numbers carefully! A common mistake in the example above would be writing the second step as $-2^2 \cdot (-3)$ and therefore ending up with $-4 \cdot -3 = 12$ as your final answer.

Practice Problems

Evaluate the following.

1)
$$x^2y$$
 where $x = 3$ and $y = 2$
2) x^2y where $x = -4$ and $y = \frac{1}{2}$
3) $-x^3$ where $x = -2$
4) $(xyz)^2$ where $x = 2, y = -1$, and $z = 4$
5) $y + \sqrt{x}$ where $x = 25$ and $y = -4$
6) $xy - \frac{3}{\sqrt{y}}$ where $x = 1$ and $y = 4$
7) $\frac{x^2}{4} - \frac{y^2}{3}$ where $x = 3$ and $y = 2$
8) $\frac{5(x+h)-5(x)}{h}$ where $x = 3$ and $h = 4$
9) $\sqrt{\frac{s+r}{|s-r|}}$ where $r = 10$ and $s = 6$
10) $\frac{-b+\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}$ where $a = 3, b = 4$, and $c = 1$

Answers

1)	18	$x^2y = (3)^2(2) = 9(2) = 18$
2)	8	$x^{2}y = (-4)^{2}\left(\frac{1}{2}\right) = 16\left(\frac{1}{2}\right) = 8$
3)	8	$-x^3 = -(-2)^3 = -(-8) = 8$
4)	64	$(xyz)^2 = (2 \cdot -1 \cdot 4)^2 = (-8)^2 = 64$
5)	1	$y + \sqrt{x} = -4 + \sqrt{25} = -4 + 5 = 1$
6)	$\frac{5}{2}$	$xy - \frac{3}{\sqrt{y}} = (1)(4) - \frac{3}{\sqrt{4}} = 4 - \frac{3}{2} = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}$
7)	$\frac{11}{12}$	$\frac{x^2}{4} - \frac{y^2}{3} = \frac{(3)^2}{4} - \frac{(2)^2}{3} = \frac{9}{4} - \frac{4}{3} = \frac{27}{12} - \frac{16}{12} = \frac{11}{12}$
8)	5	$\frac{5(x+h)-5(x)}{h} = \frac{5(3+4)-5(3)}{4} = \frac{5(7)-15}{4} = \frac{35-15}{4} = \frac{20}{4} = 5$
9)	2	$\sqrt{\frac{s+r}{ s-r }} = \sqrt{\frac{6+10}{ 6-10 }} = \sqrt{\frac{16}{ -4 }} = \sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}} = \frac{4}{2} = 2$
10)	$-\frac{1}{3}$	$\frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-4 + \sqrt{(4)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \frac{-4 + \sqrt{16 - 4 \cdot 3 \cdot 1}}{6}$
		$=\frac{-4+\sqrt{16-12}}{6}=\frac{-4+\sqrt{4}}{6}=\frac{-4+2}{6}=\frac{-2}{6}=-\frac{1}{3}$



Ratios and Proportional Relationships

2.1 Rules of Fractions

Fractions can be reduced, added, subtracted, multiplied, and divided.

Terminology

The top part of a fraction is called the *numerator* and the bottom part of a fraction is called the *denominator*.

numerator denominator

Multiplication

Multiplication is the simplest operation to perform on two fractions.

Rule

$\frac{a}{c} \cdot \frac{c}{c} - \frac{a \cdot c}{c}$	2 1 2 * 1 2	_ 1
$b^{+}d^{-}b*d$	$\frac{1}{3} * \frac{1}{2} = \frac{1}{3 * 2} = \frac{1}{6}$	$=\overline{3}$

Example

Division

Division is the second simplest operation to perform on two fractions.

Rule

	•
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c} = \frac{a * d}{b * c}$	$\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} * \frac{2}{1} = \frac{2 * 2}{3 * 1} = \frac{4}{3}$

Example

Addition and Subtraction

To add or subtract fractions, we find a common denominator. We can do this by multiplying the top and bottom of both fractions by the denominator of the other. Once the 2 fractions have the same denominator, you can add or subtract the numerators normally. The rule and example are on the next page.

Addition and Subtraction

 Rule
 Example

 $\frac{a}{b} + \frac{c}{d} = \left(\frac{d}{d}\right)\frac{a}{b} + \frac{c}{d}\left(\frac{b}{b}\right) = \frac{a*d+b*c}{b*d}$ $\frac{2}{3} + \frac{1}{2} = \left(\frac{2}{2}\right)\frac{2}{3} + \frac{1}{2}\left(\frac{3}{3}\right) = \frac{2*2+1*3}{2*3}$
 $= \frac{4+3}{6} = \frac{7}{6}$

A note about whole numbers

Whole numbers can be rewritten as fractions in order to more easily follow the formulas above.

$$2 * \frac{3}{5} = \frac{2}{1} * \frac{3}{5}$$

Practice Problems

1)	$\frac{1}{2} \cdot \frac{3}{7}$	7)	$\frac{3}{7} + \frac{2}{3}$
2)	$\frac{9}{10} \cdot \frac{2}{10}$	8)	$\frac{3}{7} - \frac{2}{3}$
3)	$\frac{-3}{5} \cdot \frac{-15}{9}$	9)	$\frac{5}{4} + \frac{5}{3}$
4)	$\frac{1}{3} \div \frac{1}{9}$	10)	$\frac{4}{9} - \frac{3}{2}$
5)	$\frac{3}{4} \div \frac{1}{2}$	11)	$\frac{5}{4} + \frac{7}{8} \left(\frac{8}{7}\right)$
6)	$8\left(\frac{1}{5}\right)$		

Answers

1)	$\frac{3}{14}$	$\frac{1}{2} \cdot \frac{3}{7} = \frac{1 \cdot 3}{2 \cdot 7} = \frac{3}{14}$
2)	$\frac{9}{50}$	$\frac{9}{10} \cdot \frac{2}{10} = \frac{9 \cdot 2}{10 \cdot 10} = \frac{18}{100} = \frac{9}{50}$
3)	1	$\frac{-3}{5} \cdot \frac{-15}{9} = \frac{-3 \cdot -15}{5 \cdot 9} = \frac{45}{45} = 1$
4)	3	$\frac{1}{3} \div \frac{1}{9} = \frac{1}{3} \cdot \frac{9}{1} = \frac{1 \cdot 9}{3 \cdot 1} = \frac{9}{3} = 3$
5)	$\frac{3}{2}$	$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \cdot \frac{2}{1} = \frac{3 \cdot 2}{4 \cdot 1} = \frac{6}{4} = \frac{3}{2}$
6)	8 5	$8\left(\frac{1}{5}\right) = \frac{8}{1} \cdot \frac{1}{5} = \frac{8 \cdot 1}{1 \cdot 5} = \frac{8}{5}$
7)	$\frac{23}{21}$	$\frac{3}{7} + \frac{2}{3} = \left(\frac{3}{3}\right)\frac{3}{7} + \frac{2}{3}\left(\frac{7}{7}\right) = \frac{3\cdot3}{3\cdot7} + \frac{2\cdot7}{3\cdot7} = \frac{9}{21} + \frac{14}{21} = \frac{9+14}{21} = \frac{23}{21}$
8)	$-\frac{5}{21}$	$\frac{3}{7} - \frac{2}{3} = \left(\frac{3}{3}\right)\frac{3}{7} - \frac{2}{3}\left(\frac{7}{7}\right) = \frac{3 \cdot 3}{3 \cdot 7} - \frac{2 \cdot 7}{3 \cdot 7} = \frac{9}{21} - \frac{14}{21} = \frac{9 - 14}{21} = -\frac{5}{21}$
9)	$\frac{35}{12}$	$\frac{5}{4} + \frac{5}{3} = \left(\frac{3}{3}\right)\frac{5}{4} + \frac{5}{3}\left(\frac{4}{4}\right) = \frac{3\cdot5}{3\cdot4} + \frac{5\cdot4}{3\cdot4} = \frac{15}{12} + \frac{20}{12} = \frac{15+20}{12} = \frac{35}{12}$
10)	$-\frac{19}{18}$	$\frac{4}{9} - \frac{3}{2} = \left(\frac{2}{2}\right)\frac{4}{9} - \frac{3}{2}\left(\frac{9}{9}\right) = \frac{2 \cdot 4}{2 \cdot 9} - \frac{3 \cdot 9}{2 \cdot 9} = \frac{8}{18} - \frac{27}{18} = \frac{8 - 27}{18} = -\frac{19}{18}$
11)	$\frac{9}{4}$	$\frac{5}{4} + \frac{7}{8} \left(\frac{8}{7}\right) = \frac{5}{4} + \frac{7 \cdot 8}{8 \cdot 7} = \frac{5}{4} + \frac{56}{56} = \left(\frac{14}{14}\right) \frac{5}{4} + \frac{56}{56} = \frac{70}{56} + \frac{56}{56} = \frac{70 + 56}{56} = \frac{126}{56} = \frac{9}{4}$

 $\frac{56}{56}$ can also be written as 1. So this problem could be simplified from this step as $\frac{5}{4} + 1$.

2.2 Mixed Fractions

The following are four definitions in regards to fractions.

$\frac{anynumber}{1} \to \frac{4}{1} = 4$
$\frac{numerator\ smaller}{denominator\ larger} \to \frac{2}{3}$
$\frac{numerator\ larger}{denominator\ smaller} \rightarrow \frac{7}{2}$
whole number $\frac{numerator\ smaller}{denominator\ larger}$
$\rightarrow 7\frac{2}{3}$

Write Improper Fraction as Mixed Fraction	
Starting point:	$\frac{7}{2}$
Re-write as the sum of fractions:	$= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2}$
Simplify to whole numbers:	$= 1 + 1 + 1 + \frac{1}{2}$
Combine:	$= 3 + \frac{1}{2}$
Write the whole number next to the proper fraction:	$= 3\frac{1}{2}$

Write Mixed Fraction as Improper Fraction

Starting point:	$2\frac{3}{7}$
Re-write as the sum of fraction and whole number:	$= 2 + \frac{3}{7}$
Expand:	$= 1 + 1 + \frac{3}{7}$
Write whole numbers as fractions:	$=\frac{7}{7}+\frac{7}{7}+\frac{3}{7}$
Add across the top $(7 + 7 + 3)$ and keep the bottom the same:	$=\frac{17}{7}$

Practice Problems

Write the following improper fractions as mixed fractions.



Write the following mixed fractions as improper fractions.



Answers

1)	$1\frac{1}{7}$	$\frac{8}{7} = \frac{7}{7} + \frac{1}{7} = 1 + \frac{1}{7} = 1\frac{1}{7}$
2)	$4\frac{1}{2}$	$\frac{9}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} = 1 + 1 + 1 + 1 + \frac{1}{2} = 4 + \frac{1}{2} = 4\frac{1}{2}$
3)	$1\frac{1}{2}$	$\frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1 + \frac{1}{2} = 1\frac{1}{2}$
4)	$3\frac{2}{3}$	$\frac{11}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3} = 1 + 1 + 1 + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}$
5)	$7\frac{2}{3}$	$\frac{23}{3} = \frac{3}{3} + \frac{2}{3} = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + \frac{2}{3} = 7 + \frac{2}{3} = 7\frac{2}{3}$
6)	$3\frac{2}{5}$	$\frac{17}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = 1 + 1 + 1 + \frac{2}{5} = 3 + \frac{2}{5} = 3\frac{2}{5}$
7)	$2\frac{5}{6}$	$\frac{17}{6} = \frac{6}{6} + \frac{6}{6} + \frac{5}{6} = 1 + 1 + \frac{5}{6} = 2 + \frac{5}{6} = 2\frac{5}{6}$
8)	3	$\frac{12}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} = 1 + 1 + 1 = 3 alternatively \frac{12}{4} = 12 \div 4 = 3$
9)	$\frac{3}{2}$	$1\frac{1}{2} = 1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$
10)	$\frac{13}{3}$	$4\frac{1}{3} = 4 + \frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} + \frac{1}{3} = \frac{13}{3}$
11)	$\frac{33}{10}$	$3\frac{3}{10} = 3 + \frac{3}{10} = \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{3}{10} = \frac{33}{10}$
12)	$\frac{13}{2}$	$6\frac{1}{2} = 6 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{13}{2}$
13)	$\frac{23}{9}$	$2\frac{5}{9} = 2 + \frac{5}{9} = \frac{9}{9} + \frac{9}{9} + \frac{5}{9} = \frac{23}{9}$
14)	$\frac{23}{3}$	$7\frac{2}{3} = 7 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{23}{3}$
15)	$\frac{11}{7}$	$1\frac{4}{7} = 1 + \frac{4}{7} = \frac{7}{7} + \frac{4}{7} = \frac{11}{7}$
2.3 Ratios and Proportions

A ratio is a way to compare two quantities by using division.

There are three dogs for every one cat	The ratio of dogs to cats is 3:1
The first quantity is placed on top of the fraction	3:1 can also be written as $\frac{3}{1}$

You can also have a ratio of units of measurement.

5	5 miles
5 miles per nour is equivalent to	1 hour

Proportions

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal. $\frac{1}{2} = \frac{2}{4}$ is an example of a proportion. When one of the four numbers in a proportion is unknown, mathematical manipulation can be used to find the missing value.

Example	
Solve for x.	
Starting point:	$\frac{x}{3} = \frac{4}{6}$
Multiply both sides by 3	$x = \left(\frac{4}{6}\right)3$
Reduce	x = 2

Note

In the last example, you can check your answer by plugging in 2 for x in the original proportion. When you reduce $\frac{4}{6}$, the equality becomes true.

$$\frac{x}{3} = \frac{4}{6} \iff \frac{2}{3} = \frac{4}{6} \iff \frac{2}{3} = \frac{2}{3} \checkmark$$

Proportions are helpful when converting units of measurement.

Units of Measurement Example

Equaling the ratios to each other gives you a solvable proportion for the unknown variable.

There are 12 inches in 1 foot. How many inches are in 2 feet?

12 inches1 foot	12 inches for every 1 foot
? inches 2 feet	We are looking for the question mark
$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{? \text{ inches}}{2 \text{ feet}}$	Place units on the same side of the divide line
$? = 2 feet * \left(\frac{12 inches}{1 foot}\right)$	Multiply each side by 2 feet
? = 2 * (12 inches)	Units cancel just as numbers do
? = 24 inches	Our final answer

Ratios are also good at transforming units of measurements from one form to another.

Units of Measurement Example

You can multiply each ratio arranged so that the units of measurements cancel, leaving the desired unit.

Convert 10 miles per hour into meters per second,

(1 mile = 1609.34 meters)

$rac{10 miles}{1 hour} *$	$\frac{1609.34 \text{ meters}}{1 \text{ mile}} * \frac{1609.34 \text{ meters}}{600000000000000000000000000000000000$	$\frac{1 \text{ hour}}{0 \text{ seconds}} = \frac{16}{2}$	6093.4 meters 60 seconds	
10 mil	es _ 1609.34 meters	1 hour	16093.4 meters	
1 hou	r * <u>1 mile</u>	* 60 seconds	60 seconds	

Practice Problems

1) What is the ratio of five nails for every hammer?

A) 5:1	C) 1:5
B) 5:5	D) 1:1

2) What is the ratio of 4 bears for every 13 beavers?

A) 13:4	C) 4:17
---------	---------

B) 4:13 D) 13:17

3) What is the people to dog ratio if 12:17?

A)
$$\frac{17 \ dogs}{12 \ people}$$
C) $\frac{12 \ dogs}{17 \ people}$ B) $\frac{12 \ people}{17 \ dogs}$ D) $\frac{17 \ people}{12 \ dogs}$

4) Find x when $\frac{x}{12} = \frac{2}{3}$

5) Find x when $\frac{x}{6} = \frac{2}{9}$

A) 1.5 C) 12
B)
$$\frac{3}{4}$$
 D) $\frac{4}{3}$

6) Find x when $\frac{8}{x} = \frac{3}{4}$

A) 12 B) 8 D) $\frac{32}{32}$ 7) Find x when $\frac{6}{9} = \frac{3}{x}$

A)
$$\frac{9}{2}$$

B) $\frac{2}{9}$
C) 18
D) $\frac{1}{18}$

8) If 12 *inches* = 1 *foot*, then how many inches are in 3 feet?

A) 24	C) $\frac{1}{36}$
B) 36	D) $\frac{1}{4}$

9) A car is driving 25 miles per hour. How many feet per second is that equivalent to if 1 mile = 5280 ft?

A) 0.000189	C) 2200
B) 132000	D) $\frac{110}{3}$

10) A square is 3 ft in length by 4 ft in height. An artist wants to scale the box down to 1 foot in length. How tall should the artist draw the square in inches to keep the original proportion?

A) $\frac{4}{3}$ inches	C) 3 feet
B) 16 inches	D) 4 inches

Answers

1)	Α
2)	В
3)	В
4)	Α
5)	D
6)	D
7)	A
8)	В
9)	D
10)	A



3.1 Rules of Exponents

An exponent is a base raised to a power. A base 'a' raised to the power of 'n' is equal to the multiplication of a, n times:

 $a^n = a \times a \times ... \times a$ [*n* times]

a is the base and n is the exponent.

Examples

$x^1 = x$	$2^1 = 2$
$x^2 = x \cdot x$	$2^2 = 2 \cdot 2$
$x^3 = x \cdot x \cdot x$	$2^3 = 2 \cdot 2 \cdot 2$
$x^4 = x \cdot x \cdot x \cdot x$	$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$
$x^5 = x \cdot x \cdot x \cdot x \cdot x$	$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Rules

$a^m \cdot a^n = a^{m+n}$
$a^m \cdot b^m = (a \cdot b)^m$
$\frac{a^m}{a^n} = a^{m-n}$
$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$(a \cdot b)^m = a^m \cdot b^m$
$(a^m)^n = a^{m \cdot n}$
$a^{-n} = \frac{1}{a^n}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$a^0 = 1$
$0^n = 0, \qquad for \ n > 0$

Product (Multiplication) Rule

When multiplying like bases, we add the powers. When multiplying like powers, we multiply the bases.

Rules	Examples
$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^5 = x^{3+5} = x^8$
	$2^3 \cdot 2^4 = 2^{3+4} = 128$
$a^m \cdot b^m = (a \cdot b)^m$	$x^2 \cdot y^2 = (x \cdot y)^2$
	$3^2 \cdot 4^2 = (3 \cdot 4)^2 = 144$
	$(2 \cdot 3)^3 = 2^3 \cdot 3^3 = 8 \cdot 27 = 216$

Quotient (Division or Fraction) Rule

When we divide like bases with different powers, subtract the bottom power from the top. When we divide like powers with different bases, divide the top base by the bottom base. Or, vice versa.

Rules	Examples
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^5}{x^3} = x^{5-3} = x^2$
	$\frac{2^3}{2^3} = 2^{5-3} = 2^2 = 4$
$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$
	$\frac{4^2}{2^2} = \left(\frac{4}{2}\right)^2 = \left(\frac{2 \cdot 2}{2}\right)^2 = 2^2 = 4$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{4}{2}\right)^2 = \frac{4^2}{2^2} = \frac{4 \cdot 4}{2 \cdot 2} = 4$

Power Rules

When raising parenthesis to a power, all elements are raised to that power. When raising a power to another power, we multiply the powers.

Rules	Examples
$(a \cdot b)^m = a^m \cdot b^m$	$(x \cdot y)^2 = x^2 \cdot y^2$ (2 \cdot 3)^2 = 2 ² \cdot 3 ² = 4 \cdot 9 = 36
$(a^m)^n = a^{m*n}$	$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$

Negative Rules

A number to a negative power is equal to one divided by that number and power. A fraction raised to a negative power is flipped.

Rules	Examples
$a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$	$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{3 \cdot 7}{2 \cdot 2} = \frac{9}{4}$

Zero Rule

Any number to the power of 0 is equal to 1. Zero to the power of any number is 0 (for numbers greater than 0).

Rules		Examples
$a^0 = 1$		$3^0 = 1$
$0^n = 0$,	for $n > 0$	$0^3 = 0$

Scientific Notation

A number written in scientific notation is written as a number between 1 and 10 multiplied by 10 raised to some exponent.

Examples

1,000	$= 1 \times 10^3$	0.01	$= 1 \times 10^{-2}$
2,000	$= 2 \times 10^3$	0.00036	$= 3.6 \text{ x } 10^4$
300,000,000	$= 3 \times 10^8$	-0.00036	$= -3.6 \times 10^4$
120	$= 1.2 \text{ x } 10^2$	-120	$= -1.2 \ge 10^2$

Practice Problems

Simplify the following expressions.

1)	$2^3 + 3^2$	7)	$\frac{2^5}{(-2)^2} + \frac{2}{2^3}$
2)	$\frac{(3^2)^3 - (2^2)^2}{31}$	8)	$(2^3)^{-2}(2^2 \cdot 1^3)^2$
3)	$3^2 \cdot 3^{-2}$	9)	$\frac{x^5}{x^2}$
4)	$-2^4 + (-2)^4 - 2^1$	10)	$\frac{x^3y^2}{x^2+5}$
5)	$\left(\frac{3}{2}\right)^3 + \frac{\left(3^2\right)^2 - 50}{8}$	11)	$(2x^2y^3)^0$
6)	$(3^2 \cdot 2^3) \cdot 3^{-3} + (3 \div 3^3)$	12)	$(3x^4y^{-2})^{-3}$
Iany	of these problems can be solved in	12)	$(2x^3y^2)^{-2}$

Many of these problems can be solved in multiple ways. Try using the definition of exponents to expand the numbers or variables, or try solving them using different rules of exponents!

(More problems on next page)

Convert the following numbers to scientific notation.

- 13) 0.004695
- 14) 9.81
- 15) 8,679,000

Convert the following numbers to standard form.

- 16) 345.6 \times 10⁴
- 17) -64.7×10^4
- 18) 5.4×10^{-2}

$2^3 + 3^2 = 8 + 9 = 17$ 1) 17 $\frac{\left(3^2\right)^3 - \left(2^2\right)^2}{31} = \frac{3^6 - 2^4}{31} = \frac{729 - 16}{31} = \frac{713}{31} = 23$ 2) 23 $3^2 \cdot 3^{-2} = 3^2 \cdot \frac{1}{3^2} = \frac{3^2}{3^2} = 1$ or $3^2 \cdot 3^{-2} = 3^{2-2} = 3^0 = 1$ 3) 1 $-2^4 + (-2)^4 - 2^1 = -16 + 16 - 2 = 0 - 2 = -2$ 4) -2 $\frac{29}{4}$ $\left(\frac{3}{2}\right)^3 + \frac{\left(3^2\right)^2 - 50}{8} = \frac{3^3}{2^3} + \frac{3^4 - 50}{8} = \frac{27}{8} + \frac{81 - 50}{8} = \frac{27}{8} + \frac{31}{8} = \frac{27 + 31}{8} = \frac{58}{8} = \frac{29}{4}$ 5) 25 $(3^2 \cdot 2^3) \cdot 3^{-3} + (3 \div 3^3) = (9 \cdot 8) \cdot \frac{1}{3^3} + \left(\frac{3}{3^3}\right) = \frac{72}{27} + \frac{3}{27} = \frac{75}{27} = \frac{25}{9}$ 6) 9 33 $\frac{2^5}{(-2)^2} + \frac{2}{2^3} = \frac{32}{4} + \frac{2}{8} = \binom{2}{2}\frac{32}{4} + \frac{2}{8} = \frac{64}{8} + \frac{2}{8} = \frac{64+2}{8} = \frac{64+2}{8} = \frac{66}{8} = \frac{33}{4}$ 7) 4 $\frac{1}{4}$ $(2^3)^{-2}(2^2 \cdot 1^3)^2 = 2^{-6}(4 \cdot 1)^2 = \frac{1}{2^6} \cdot (4)^2 = \frac{16}{64} = \frac{1}{4}$ 8) $\frac{x^{5}}{x^{2}} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{x \cdot x \cdot x}{1} = x^{3} \quad or \quad \frac{x^{5}}{x^{2}} = x^{5-2} = x^{3}$ x^3 9) $\frac{x}{v^3}$ $\frac{x^3y^2}{x^2y^5} = \frac{x \cdot x \cdot x \cdot y \cdot y}{x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y} = \frac{x}{y \cdot y \cdot y} = \frac{x}{y^3} \quad or \quad \frac{x^3y^2}{x^2y^5} = x^{3-2}y^{2-5} = x^1y^{-3} = x\left(\frac{1}{y^3}\right) = \frac{x}{y^3}$ 10) $(2x^2y^3)^0 = 2^0x^0y^0 = 1 \cdot 1 \cdot 1 = 1$ or $(2x^2y^3)^0 = 1$ 11) 1 (anything to the power of zero is one) $\frac{4y^{10}}{27x^6}$ $\frac{\left(3x^4y^{-2}\right)^{-3}}{\left(2x^3y^2\right)^{-2}} = \frac{3^{-3}x^{-12}y^6}{2^{-2}x^{-6}y^{-4}} = \frac{2^2x^6y^4y^6}{3^3x^{12}} = \frac{2^2x^6y^{10}}{3^3x^{12}} = \frac{4x^6y^{10}}{27x^{12}} = \frac{4y^{10}}{27x^{6}}$ 12) 13) 4.695×10^{-3} 9.81×10^{0} 14) 8.679×10^{6} 15) 3456000 16) 17) -64700018) 0.054

Answers

3.2 Simplifying Radicals

A radical is anything with the root symbol $\sqrt{}$. Radicals are **fractional exponents**. The notation is very different, but they follow the same basic rules as regular exponents! With practice, you will be able to see the similarities. The following shows how radicals work on a basic level and how to simplify them.

Notation

$\sqrt{x} = \sqrt[2]{x} = x^{1/2}$	$4^{1/2} = \sqrt[2]{4} = \sqrt[2]{2 \cdot 2} = 2$
$\sqrt[3]{x} = x^{1/3}$	$8^{1/3} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2$
$\sqrt[4]{x} = x^{1/4}$	$16^{1/4} = \sqrt[4]{16} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = 2$
$\sqrt[5]{x} = x^{1/5}$	$32^{1/5} = \sqrt[5]{32} = \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2$
$\sqrt[6]{x} = x^{1/6}$	$64^{1/6} = \sqrt[6]{64} = \sqrt[6]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2$

Simplification Example

A radical is fully simplified when the radicand (the part under the radical) has no square factors. For instance, $\sqrt{40} = \sqrt{4 \cdot 5 \cdot 2}$, and since 4 is a square number, $\sqrt{40}$ is not simplified. From here, use the rules of radicals to split the root apart and simplify:

$$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$$

So $2\sqrt{10}$ is the most simplified form of $\sqrt{40}$.

Pattern-Based Approach

Another way to simplify is to factor the radicand completely. For example, $\sqrt{40} = \sqrt{2 \cdot 2 \cdot 2 \cdot 5}$. Notice there are three twos. The type of radical we are using is a square root ($\sqrt{2 \cdot 2 \cdot 2 \cdot 5} = \sqrt[2]{2 \cdot 2 \cdot 2 \cdot 5}$). Since that *two* is there, we find pairs of *two* and represent the two outside, leaving behind the unpaired numbers under the radical. $\sqrt[2]{2 \cdot 2 \cdot 2 \cdot 5} = 2 \cdot \sqrt[2]{2 \cdot 5}$. There are more examples on the next page.

$\sqrt{40} = \sqrt{2 \cdot 2 \cdot 2 \cdot 5} = 2 \cdot \sqrt{2 \cdot 5} = 2\sqrt{10}$	groups of 2 because of $$
$\sqrt{72} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 2} = 2 \cdot 3 \cdot \sqrt{2} = 6\sqrt{2}$	groups of 2 because of $$
$\sqrt{16} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2} = 2 \cdot 2 = 4$	groups of 2 because of $$
$\sqrt[3]{40} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5} = 2 \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$	groups of 3 because of $\sqrt[3]{}$
$\sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2 = 2$	groups of 3 because of $\sqrt[3]{}$
$\sqrt[3]{250} = \sqrt[3]{5 \cdot 5 \cdot 5 \cdot 2} = 5 \cdot \sqrt[3]{2} = 5\sqrt[3]{2}$	groups of 3 because of $\sqrt[3]{}$
$\sqrt[4]{16} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = 2 = 2$	groups of 4 because of $\sqrt[4]{}$
$\sqrt[4]{32} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2 \cdot \sqrt[4]{2} = 2\sqrt[4]{2}$	groups of 4 because of $\sqrt[4]{}$

If you factor the radicand completely and can't find any pairs, then the radical is already as simple as it can be. For example, $\sqrt{30} = \sqrt{2 \cdot 3 \cdot 5}$. Since there are no pairs, $\sqrt{30}$ is the most simplified form.

Practice Problems

Simplify the following radicals.

1) \(\sqrt{20}\)	9) \[\]
2) $\sqrt{16}$	10) \[\]
3) \{\24	11) $\sqrt{48}$
4) $\sqrt{18}$	12) $\sqrt{300}$
5) $\sqrt{180}$	13) $\sqrt{150}$
6) $\sqrt{12}$	14) \{\80
7) \{\sqrt{28}}	$15)\frac{\sqrt{20}}{1}$
8) $\sqrt{50}$	· 2

Answers

1)	$2\sqrt{5}$	$\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$
2)	4	$\sqrt{16} = \sqrt{4 \cdot 4} = 4$
3)	$2\sqrt{6}$	$\sqrt{24} = \sqrt{3 \cdot 2 \cdot 2 \cdot 2} = 2\sqrt{6}$
4)	$3\sqrt{2}$	$\sqrt{18} = \sqrt{3 \cdot 3 \cdot 2} = 3\sqrt{2}$
5)	6√5	$\sqrt{180} = \sqrt{3 \cdot 3 \cdot 2 \cdot 2 \cdot 5} = 6\sqrt{5}$
6)	$2\sqrt{3}$	$\sqrt{12} = \sqrt{3 \cdot 2 \cdot 2} = 2\sqrt{3}$
7)	$2\sqrt{7}$	$\sqrt{28} = \sqrt{2 \cdot 2 \cdot 7} = 2\sqrt{7}$
8)	$5\sqrt{2}$	$\sqrt{50} = \sqrt{2 \cdot 5 \cdot 5} = 5\sqrt{2}$
9)	$3\sqrt{5}$	$\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5} = 3\sqrt{5}$
10)	7√2	$\sqrt{98} = \sqrt{2 \cdot 7 \cdot 7} = 7\sqrt{2}$
11)	$4\sqrt{3}$	$\sqrt{48} = \sqrt{4 \cdot 4 \cdot 3} = 4\sqrt{3}$
12)	$10\sqrt{3}$	$\sqrt{300} = \sqrt{10 \cdot 10 \cdot 3} = 10\sqrt{3}$
13)	$5\sqrt{6}$	$\sqrt{150} = \sqrt{5 \cdot 5 \cdot 3 \cdot 2} = 5\sqrt{6}$
14)	$4\sqrt{5}$	$\sqrt{80} = \sqrt{4 \cdot 4 \cdot 5} = 4\sqrt{5}$
15)	$\sqrt{5}$	$\frac{\sqrt{20}}{2} = \frac{\sqrt{2 \cdot 2 \cdot 5}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

3.3 Rules of Radicals

Since radicals are **fractional exponents**, we can use rules of exponents to manipulate them. The following are some of the properties and notation with radicals.

Rules

$x^{rac{a}{b}}=\sqrt[b]{x^{a}}$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$
$\sqrt[a]{x^a} = x$	$\sqrt[2]{x^2} = x^{\frac{2}{2}} = x^1 = x$
$\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{x \cdot y}$	$\sqrt[3]{x} \cdot \sqrt[3]{y} = \sqrt[3]{xy}$
$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$	$\sqrt[5]{\frac{x}{y}} = \frac{\sqrt[5]{x}}{\sqrt[5]{y}}$

Fractional Exponent Example

Use the first rule to change the form into a radical.

Starting Point:	$2^{\frac{4}{3}}$
Change into radical form:	$\sqrt[3]{2^4}$
Evaluate:	³ √16
Simplify:	$2\sqrt[3]{2}$

Fractional Exponent Example

Use the first rule to change the form into a radical.

Starting Point:	$\sqrt[3]{2^3}$
Change into radical form:	$2^{\frac{3}{3}}$
$2^{\frac{3}{3}} = 2^1 = 2$, so:	2

Multiplication Rule

When two radicals of the same type are multiplied together, they can be combined under one radical and vice versa.

Rules	Examples
$\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy}$	$\sqrt{2} \cdot \sqrt{3} = \sqrt{2 \cdot 3} = \sqrt{6}$
$\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$	$\sqrt{10} = \sqrt{2 \cdot 5} = \sqrt{2} \cdot \sqrt{5}$

Division Rule

When two radicals of the same type are divided, they can be combined under one radical and vice versa.

Rules	Examples
$n \boxed{\frac{x}{x} = \frac{\sqrt[n]{x}}{\sqrt{x}}}$	$2 \sqrt{2}$
$\sqrt{y} \sqrt{\frac{n}{\sqrt{y}}}$	$\sqrt{3} \sqrt{3}$
$\frac{\sqrt[n]{x}}{\sqrt[n]{x}} = \sqrt[n]{\frac{x}{x}}$	$\sqrt{2}$ 2
$\sqrt[n]{y} \sqrt{y} \sqrt{y}$	$\overline{\sqrt{5}} = \sqrt{5}$

Note On Multiplication And Division

If the radicals are not the same type (number), the multiplication and division rules do not apply.

Rule	Examples
$\sqrt[a]{x} \cdot \sqrt[b]{y}$ cannot combine	$\sqrt[2]{x} \cdot \sqrt[3]{y}$ is already as simple as possible
$\frac{\sqrt[a]{x}}{\sqrt[b]{y}}$ cannot combine	$\frac{\sqrt[5]{x}}{\sqrt[4]{y}}$ is already as simple as possible

Exponent Example

Since radicals are exponents, we can combine our knowledge of exponents and radicals to evaluate them.

Starting Point:	$\sqrt[3]{2^3}$
Change radical into exponent form:	$(2^3)^{\frac{1}{3}}$
Use rules of exponents:	$(2)^{\frac{3}{3}}$
Simplify exponent:	2 ¹
Use rules of exponents:	2

Practice Problems

Evaluate the following radicals.

1)
$$(\sqrt{3})^2$$

2) $\sqrt{(3)^2}$
3) $\sqrt{2} \cdot \sqrt{15}$
4) $\sqrt{6} \cdot \sqrt{8}$
5) $\sqrt{\frac{4}{9}}$
6) $-\sqrt{\frac{6}{36}}$
7) $16^{\frac{2}{8}}$
8) $(\sqrt[3]{8})^2$
9) $\frac{8}{3\sqrt{64}}$
10) $\sqrt{7} + 5\sqrt{7} - 2\sqrt{7}$
11) $\sqrt[3]{-125} - \sqrt[4]{81}$
12) $\frac{1}{3}(2\sqrt{2} + 6\sqrt{2} - 5\sqrt{2} + \sqrt{5} + 2\sqrt{5})$

Answers

1)	3	$\left(\sqrt{3}\right)^2 = 3^{\frac{2}{2}} = 3^1 = 3$
2)	3	$\sqrt{(3)^2} = 3^{\frac{2}{2}} = 3^1 = 3$
3)	$\sqrt{30}$	$\sqrt{2} \cdot \sqrt{15} = \sqrt{30}$
4)	$4\sqrt{3}$	$\sqrt{6} \cdot \sqrt{8} = \sqrt{6 \cdot 8} = \sqrt{48} = 4\sqrt{3}$
5)	$\frac{2}{3}$	$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$
6)	$-\frac{\sqrt{6}}{6}$	$-\sqrt{\frac{6}{36}} = -\frac{\sqrt{6}}{\sqrt{36}} = -\frac{\sqrt{6}}{6}$
7)	2	$16^{\frac{2}{8}} = \sqrt[8]{(16)^2} = \sqrt[8]{16 \cdot 16} = \sqrt[8]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2$
8)	4	$\left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$
9)	$\frac{1}{3}$	$\frac{8}{3\sqrt{64}} = \frac{8}{3 \cdot 8} = \frac{1}{3}$
10)	$4\sqrt{7}$	$\sqrt{7} + 5\sqrt{7} - 2\sqrt{7} = 6\sqrt{7} - 2\sqrt{7} = 4\sqrt{7}$
11)	-8	$\sqrt[3]{-125} - \sqrt[4]{81} = -5 - 3 = -8$
12)	$\sqrt{2} + \sqrt{5}$	$\frac{1}{3}(2\sqrt{2} + 6\sqrt{2} - 5\sqrt{2} + \sqrt{5} + 2\sqrt{5}) = \frac{1}{3}(8\sqrt{2} - 5\sqrt{2} + \sqrt{5} + 2\sqrt{5})$
		$= \frac{1}{3} \left(3\sqrt{2} + \sqrt{5} + 2\sqrt{5} \right) = \frac{1}{3} \left(3\sqrt{2} + 3\sqrt{5} \right)$
		$=\sqrt{2}+\sqrt{5}$

3.4 Rationalizing

When radicals appear in a fraction, we may have to use algebra to rearrange the fraction so that it is in the proper form. Radicals are allowed to be in the numerator, but <u>not</u> the denominator. This is a convention that will be followed on the test.

If a radical is in the denominator, the process of rearranging the fraction so that the radical is only in the numerator is called **rationalization**. First some terminology is given, then examples of two situations involving rationalization are outlined.

Terminology

A **binomial** is formed when two terms are added or subtracted from one another. (x + y) is a binomial. (2x-7) is also a binomial.

The **conjugate** of a binomial is created when the plus or minus sign in the <u>middle</u> of the binomial is changed to the opposite sign. (x - y) is the conjugate of (x + y). (7x + 4) is the conjugate of (7x - 4).

Example

Consider the following fraction. In order to rationalize a fraction with a radical in the denominator, multiply both the numerator and denominator by that same radical.



In other examples, reduce further if necessary.

Example

When a radical in the denominator is part of a binomial, multiply the numerator and denominator by the conjugate.



Practice Problems

Rationalize the following fractions.

1)
$$\frac{1}{\sqrt{2}}$$

2) $\frac{1}{\sqrt{3}}$
3) $\frac{2}{\sqrt{5}}$
4) $\frac{4}{\sqrt{3}}$
5) $\frac{x}{\sqrt{2}}$

6)
$$\frac{1}{3+\sqrt{2}}$$

7)
$$\frac{1}{1+\sqrt{3}}$$

8)
$$\frac{5}{2-\sqrt{2}}$$

9)
$$\frac{3}{4-\sqrt{3}}$$

10)
$$\frac{2}{x+\sqrt{5}}$$

Answers

1)	$\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{1 \cdot \sqrt{2}}{\left(\sqrt{2}\right)^2} = \frac{\sqrt{2}}{2}$
2)	$\frac{\sqrt{3}}{3}$	$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{1 \cdot \sqrt{3}}{\left(\sqrt{3}\right)^2} = \frac{\sqrt{3}}{3}$
3)	$\frac{2\sqrt{5}}{5}$	$\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{2 \cdot \sqrt{5}}{\left(\sqrt{5}\right)^2} = \frac{2\sqrt{5}}{5}$
4)	$\frac{4\sqrt{3}}{3}$	$\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{4 \cdot \sqrt{3}}{\left(\sqrt{3}\right)^2} = \frac{4\sqrt{3}}{3}$
5)	$\frac{x\sqrt{2}}{2}$	$\frac{x}{\sqrt{2}} = \frac{x}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{x \cdot \sqrt{2}}{\left(\sqrt{2}\right)^2} = \frac{x\sqrt{2}}{2}$
6)	$\frac{3-\sqrt{2}}{7}$	$\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right) = \frac{3-\sqrt{2}}{3^2-\left(\sqrt{2}\right)^2} = \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$
7)	$-\frac{1-\sqrt{3}}{2}$	$\frac{1}{1+\sqrt{3}} = \frac{1}{1+\sqrt{3}} \left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right) = \frac{1-\sqrt{3}}{1^2 - \left(\sqrt{3}\right)^2} = \frac{1-\sqrt{3}}{1-3} = \frac{1-\sqrt{3}}{-2} = -\frac{1-\sqrt{3}}{2}$
8)	$\frac{10+5\sqrt{2}}{2}$	$\frac{5}{2-\sqrt{2}} = \frac{5}{2-\sqrt{2}} \left(\frac{2+\sqrt{2}}{2+\sqrt{2}}\right) = \frac{5(2+\sqrt{2})}{2^2-(\sqrt{2})^2} = \frac{10+5\sqrt{2}}{4-2} = \frac{10+5\sqrt{2}}{2}$
9)	$\frac{12+3\sqrt{3}}{13}$	$\frac{3}{4-\sqrt{3}} = \frac{3}{4-\sqrt{3}} \left(\frac{4+\sqrt{3}}{4+\sqrt{3}}\right) = \frac{3(4+\sqrt{3})}{4^2-(\sqrt{3})^2} = \frac{12+3\sqrt{3}}{16-3} = \frac{12+3\sqrt{3}}{13}$
10)	$\frac{2x - 2\sqrt{5}}{x^2 - 5}$	$\frac{2}{x+\sqrt{5}} = \frac{2}{x+\sqrt{5}} \left(\frac{x-\sqrt{5}}{x-\sqrt{5}}\right) = \frac{2(x-\sqrt{5})}{x^2-(\sqrt{5})^2} = \frac{2x-2\sqrt{5}}{x^2-5}$



Linear Expressions and Equations

4.1 Simplifying Expressions

Some algebraic expressions can be reduced to a simpler form. Most of the time, the answers to a problem will be in the simplest form.

Use the basic rules of algebra to combine like terms and simplify.

Example	
Combine like terms when possible.	
Starting Point:	$2y^3 \cdot 4y^2$
Group like terms:	$2 \cdot 4 \cdot y^3 \cdot y^2$
Combine terms that can be combined:	$8 \cdot y^{3+2}$
Simplify exponent:	8y ⁵

Example	
Combine like terms.	
Starting Point:	$x^2 + 3x^2 + 2x^2 + x^3$
Combine based on number in front:	$6x^2 + x^3$

Example

Cancel terms that are in both the numerator and denominator of a fraction!

Starting Point:	$\frac{3x^2y^4}{6x^4y^3}$
Expand the expression:	$\frac{3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y}{3 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}$
Cancel out terms that appear on the top and bottom:	$\frac{3 \cdot x \cdot x \cdot y \cdot \chi \cdot \chi \cdot \chi}{2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot \chi \cdot \chi \cdot \chi \cdot \chi}$
Write without the cancelled terms:	$\frac{y}{2 \cdot x \cdot x}$
Final answer:	$\frac{y}{2x^2}$

Example

Use the rules of exponents, radicals and fractions to simplify.

Starting Point:	$2x^{-3}$
Use rules of exponents:	$2\left(\frac{1}{x^3}\right)$
Multiply:	$\frac{2}{x^3}$

Example

Factor and use cancellation when you come across polynomials! Refer to Section 5.2 to learn more about factoring.

Starting Point:	$\frac{x^2+4x+4}{x^2-4}$
Factor the numerator and denominator:	$\frac{(x+2)(x+2)}{(x+2)(x-2)}$
Cancel out terms that appear on the top and bottom:	$\frac{(x+2)(x+2)}{(x+2)(x-2)}$
Write without the cancelled terms:	$\frac{(x+2)}{(x-2)}$
Final answer:	$\frac{x+2}{x-2}$

Practice Problems

Simplify the following algebraic expressions.

- 1) x + 3x + 4x 2x
- 2) -7 + 3x + 7 5x + 1
- 3) $3x^2 \cdot 3x^4$
- 4) $2x \cdot x^{-2}$
- 5) $x \div x^3$
- 6) $x^2 \cdot x^1 + x^3$
- 7) $4x^3 8x^3 + x^3$
- 8) $\frac{x}{2} + \frac{x}{8}$
- 9) $\frac{x^2y^7}{2x^3y^4}$

Answers

1)	6 <i>x</i>	x + 3x + 4x - 2x = 4x + 4x - 2x = 8x - 2x = 6x
2)	1 - 2x	-7 + 3x + 7 - 5x + 1 = -7 + 7 + 1 - 5x + 3x = 1 - 5x + 3x = 1 - 2x
3)	9x ⁶	$3x^2 \cdot 3x^4 = 3 \cdot 3 \cdot x^2 \cdot x^4 = 9 \cdot x^2 \cdot x^4 = 9 \cdot x^6 = 9x^6$
4)	$\frac{2}{x}$	$2x \cdot x^{-2} = 2x \cdot \frac{1}{x^2} = \frac{2x}{x^2} = \frac{2}{x}$
5)	$\frac{1}{x^2}$	$x \div x^3 = \frac{x}{x^3} = \frac{1}{x^2}$
6)	$2x^{3}$	$x^2 \cdot x^1 + x^3 = x^3 + x^3 = 2x^3$
7)	$-3x^{3}$	$4x^3 - 8x^3 + x^3 = -4x^3 + x^3 = -3x^3$
8)	$\frac{5x}{8}$	$\frac{x}{2} + \frac{x}{8} = \left(\frac{4}{4}\right)\frac{x}{2} + \frac{x}{8} = \frac{4x}{8} + \frac{x}{8} = \frac{4x+x}{8} = \frac{5x}{8}$
9)	$\frac{y^3}{2x}$	$\frac{x^2y^7}{2x^3y^4} = \frac{x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}{2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y} = \frac{y \cdot y \cdot y}{2 \cdot x} = \frac{y^3}{2x}$

4.2 Solving Linear Equations

The overall goal when solving equations is to isolate the variable on one side of the equation and send all of the numbers to the other side.

Rules

What you do to one side of the equation you must do to the other!

When you do something, it must be done to the entire side, not just one part.

To undo addition, subtract from both sides.

To undo subtraction, add to both sides.

To undo multiplication, divide on both sides.

To undo division, multiply on both sides.

Example

Consider the equation 3x + 7 = 1.

To solve for x, begin by moving single integers to the right-hand side.

	3x + 7 = 1
Subtract 7 from each side since	-7 -7
subtraction is the opposite of addition.	3x = -6

Now, divide the remaining number away from the variable.

	$\frac{3x}{3} = \frac{-6}{3}$	-	Divide both sides by 3 since division is the opposite of
Final answer! Try plugging	r = -2		multiplication.
x = -2 back into the			
original equation!			

Example

Consider the equation 2x - 3 - 2 = 3.

To solve for x, begin by combining like terms.

-3 - 2 = -5 2x - 3 - 2 = 3 2x - 5 = 3

Now proceed by moving single integers to the right-hand side.

$$2x - 5 = 3$$

$$+5 + 5$$

$$2x = 8$$
Add 5 to both sides since addition is the opposite of subtraction (the negative 5).

Now, divide the remaining number away from the variable.

$$\frac{2x}{2} = \frac{8}{2}$$
Divide both sides by 2 since division is the opposite of multiplication.
Final answer! Try plugging $x = 4$ back into the original equation!

Example

Consider the equation -7x + 1 + 2x = 9x - 8 + 1.

To solve for x, begin by combining like terms.

$$-7x + 1 + 2x = 9x - 8 + 1$$

$$-5x + 1 = 9x - 7$$

Now proceed by moving single integers to the right-hand side.

$$-5x + 1 = 9x - 7$$

$$-1 - 1$$
Make sure like terms are lined up!
$$-5x = 9x - 8$$

If we want the variable to be isolated on one side, we must now move all variables to the left-hand side.

$$-5x = 9x - 8$$
$$-9x - 9x$$
$$-14x = -8$$

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Now, divide the remaining number away from the variable.

Divide both sides by -14
since division is the opposite
of multiplication.
$$\frac{-14x}{-14} = \frac{-8}{-14}$$
$$x = \frac{4}{7}$$
Final answer! Try plug

Final answer! Try plugging $x = \frac{4}{7}$ back into the original equation!

Practice Problems

Solve the following equations for the variable.

- 1) 2x 3 = 1
- 2) 4x + 6 = 7
- 3) 14x 5 = 9
- 4) 7x + 8 = 1
- 5) 2x + 3 = 7x 1
- 6) 5x + 2 = 3x 6
- 7) x + 5 = x 6
- 8) $\frac{1}{2}x + \frac{3}{2} = 4$
- 9) $\frac{3}{4}x + 2 = x 5$
- $10)\frac{1}{2}x + \frac{3}{2}(x+1) \frac{1}{4} = 5$
- 11) $5x + 2 = \frac{1}{3}x$
- 12) 5(x+2) = 3x
- 13) $\frac{1}{5}x + \frac{1}{3} = 3$ 14) $3x + \frac{1}{2}(x + 2) = -2$
| 1) | x = 2 | $2x - 3 = 1 \rightarrow 2x = 4 \rightarrow x = 2$ |
|-----|--------------------|---|
| 2) | $x = \frac{1}{4}$ | $4x + 6 = 7 \rightarrow 4x = 1 \rightarrow x = \frac{1}{4}$ |
| 3) | <i>x</i> = 1 | $14x - 5 = 9 \rightarrow 14x = 14 \rightarrow x = 1$ |
| 4) | x = -1 | $7x + 8 = 1 \rightarrow 7x = -7 \rightarrow x = -1$ |
| 5) | $x = \frac{4}{5}$ | $2x + 3 = 7x - 1 \to 2x + 4 = 7x \to 4 = 5x \to \frac{4}{5} = x$ |
| 6) | x = -4 | $5x + 2 = 3x - 6 \rightarrow 5x = 3x - 8 \rightarrow 2x = -8 \rightarrow x = -4$ |
| 7) | no solution | $x + 5 = x - 6 \rightarrow 5 = -6$ (but since $5 \neq -6$, no solution) |
| 8) | x = 5 | $\frac{1}{2}x + \frac{3}{2} = 4 \to \frac{1}{2}x = \frac{5}{2} \to x = 5$ |
| 9) | <i>x</i> = 28 | $\frac{3}{4}x + 2 = x - 5 \to \frac{3}{4}x + 7 = x \to 7 = \frac{1}{4}x \to 28 = x$ |
| 10) | $x = \frac{15}{8}$ | $\frac{1}{2}x + \frac{3}{2}(x+1) - \frac{1}{4} = 5 \rightarrow \frac{1}{2}x + \frac{3}{2}x + \frac{3}{2} - \frac{1}{4} = 5 \rightarrow 2x + \frac{5}{4} = 5 \rightarrow 2x = \frac{15}{4} \rightarrow x = \frac{15}{8}$ |
| 11) | $x = -\frac{3}{7}$ | $5x + 2 = \frac{1}{3}x \to 5x - \frac{1}{3}x = -2 \to \frac{14}{3}x = -2 \to 14x = -6 \to x = -\frac{6}{14} \to x = -\frac{3}{7}$ |
| 12) | x = -5 | $5(x+2) = 3x \to 5x + 10 = 3x \to 10 = -2x \to -5 = x$ |
| 13) | $x = \frac{40}{3}$ | $\frac{1}{5}x + \frac{1}{3} = 3 \to \frac{1}{5}x = \frac{8}{3} \to x = \frac{40}{3}$ |
| 14) | $x = -\frac{6}{7}$ | $3x + \frac{1}{2}(x+2) = -2 \rightarrow 3x + \frac{1}{2}x + 1 = -2 \rightarrow \frac{7}{2}x + 1 = -2 \rightarrow \frac{7}{2}x = -3 \rightarrow x = -\frac{6}{7}$ |

4.3 Solving Inequalities

Solving inequalities is almost exactly the same as solving equations! The equal sign = has simply been replaced with one of the following: $\langle \leq \rangle \geq$.

However, there is one extra rule to solving inequalities. If you ever **divide or multiply by a negative number** in your solving process, you must switch the direction the inequality sign is facing.

Example

Consider the equation $3x + 7 \ge 1$.

To solve for x, begin by moving single integers to the right-hand side.

	3x + 7 > 1
Subtract 7 from each side since subtraction is the opposite of addition.	$\frac{-7 - 7}{3r > -6}$
You do not have to switch the	$5\lambda \ge 0$
direction of the \geq sign when	
subtracting. Only for multiplying or	
dividing by a negative number!	

Now, divide the remaining number away from the variable.



Example

Consider the equation $-5x - 8 \ge 2$.

To solve for x, begin by moving single integers to the right-hand side.

	$-5x - 8 \ge 2$
Add 8 to both sides since addition is the opposite of subtraction.	+8 +8
	$-5x \ge 10$

Now, divide the remaining number



The \geq sign <u>MUST</u> change direction now, since we divided by a negative number!

This is your final answer! It means that any number less than or equal to -2 will satisfy the original inequality.

Try plugging x = -2 back into the original inequality! Notice that the inequality holds $(2 \ge 2)$.

Now try plugging in other numbers less than -2! The inequality will hold!

Example

If your final answer has > or < involved (instead of \geq or \leq), this **excludes** the number on the other side of the sign.

For example, x > 5 means the answer is any number greater than 5, but **not** including 5.

Example

Solve the following inequalities.

- 1) $x 3 \ge 12$
- 2) 4*s* < −16
- 3) $5w + 2 \le -48$
- 4) 2k > 7
- 5) $2y + 4 \le y + 8$
- 6) $-3a \ge 9$
- 7) -3(z-6) > 2z-2
- 8) $-2n + 8 \le 2n$

1)	$x \ge 15$	$x - 3 \ge 12 \rightarrow x \ge 15$
2)	<i>s</i> < -4	$4s < -16 \rightarrow s < -4$
3)	$w \leq -10$	$5w + 2 \le -48 \to 5w \le -50 \to w \le -10$
4)	$k > \frac{7}{2}$	$2k > 7 \to k > \frac{7}{2}$
5)	$y \le 4$	$2y + 4 \le y + 8 \rightarrow 2y \le y + 4 \rightarrow y \le 4$
6)	$a \leq -3$	$-3a \ge 9 \rightarrow a \le -3$
7)	<i>z</i> < 4	$-3(z-6) > 2z - 2 \to -3z + 18 > 2z - 2 \to 18 > 5z - 2 \to 20 > 5z \to 4 > z$
8)	$n \ge 2$	$-2n+8 \le 2n \to 8 \le 4n \to 2 \le n$

4.4 Solving Systems of Equations

Sometimes there are two or more equations that are related because their variables are the same. This means that if you find that x = 4 in one equation, the x in the other equation is also guaranteed to be 4.

Most often you will have to create two equations from a word problem and then solve them as a system!

Example

The following is a system of equations:

2x + y = 5 and x - 2y = 15

The solution to this system is x = 5 and y = -5. Try plugging these values into each equation! They work for both!

Example

Consider the system: 2x + y = 5 and x - 2y = 15

To solve this system, we must isolate one of the variables. In this case, it looks easy to isolate the x in the second equation, so we'll start with that. (You could just as easily choose to isolate the y variable in the first equation. It doesn't make a difference).

$$x - 2y = 15$$

 $x = 2y + 15$ Use algebra here and
isolate the variable.
For more on solving
equations, refer to
Section 4.2

(Continued on next page)

Example Continued

Notice, x and 2y + 15 are exactly equal to one another. This means we can replace any x we see with 2y + 15.

We will replace the x in the first equation with 2y + 15.

```
2x + y = 52(2y + 15) + y = 54y + 30 + y = 55y + 30 = 5
```

Now that we have substituted that expression in for x, our first equation has only one variable to solve for! So now we solve for y.

$$5y + 30 = 5$$

 $5y = -25$
 $y = -5$

So y = -5! Now we can take that information and plug y = -5 in to either of our original equations in order to find x. It does not matter which equation we choose because they are both related!

```
x - 2y = 15x - 2(-5) = 15x + 10 = 15x = 5
```

So our final answer is x = 5, y = -5.

Example

Solve the following systems of equations.

- 1) 2x + y = 5 and x 2y = 15
 2) y = 2x + 4 and y = -3x + 9
 3) y = x + 3 and 42 = 4x + 2y
 4) 3x + y = 12 and y = -2x + 10
 5) y = -3x + 5 and 5x 4y = -3
 6) y = -2 and 4x 3y = 18
 7) -7x 2y = -13 and x 2y = 11
- 8) 6x 3y = 5 and y 2x = 8

1)	x = 5, y = -5
2)	x = 1, y = 6
3)	x = 6, y = 9
4)	x = 2, y = 6
5)	x = 1, y = 2
6)	x = 3, y = -2
7)	x = 3, y = -4
8)	no solution

Parallel lines never intersect, so there is no solution!

4.5 Graphing Linear Equations

There are two methods to graph a line:

1) Use the *x* and *y* intercepts: If you have the coordinates of the *x* and *y* intercepts, plot those two points and connect them with a line.



2) Use slope-intercept form: If you have an equation in slope intercept form (y=mx+b), plot the *y*-intercept and then construct the line using the slope. If the equation is not in slope-intercept form, it can be rearranged to be in slope-intercept form.





Method 1

Consider the equation 6x + 2y = 12.

Find the *x*-intercept:

To find the *x*-intercept, set *y* equal to zero and solve for *x*.

6x + 2y = 12	
6x + 2(0) = 12	
6x = 12	This is the <i>x</i> -intercept,
x = 2	meaning the point (2,0) is on
	the graph.

Find the *y*-intercept:

To find the *y*-intercept, set *x* equal to zero and solve for *y*.

$$6x + 2y = 12$$

$$6(0) + 2y = 12$$

$$2y = 12$$

$$y = 6$$

This is the *y*-intercept,
meaning the point (0,6) is on
the graph.

Now, plot the points (2,0) and (0,6) on the graph and connect them with a line:



Method 2

Consider the equation 6x + 2y = 12.

First, use algebra to rearrange the equation:

6x + 2y = 12 2y = -6x + 12 y = -3x + 6Now the equation is in yintercept form (y=mx+b).

Then identify your slope and *y*-intercept:

Compare y = -3x + 6 with y = mx + b.

In this case, m = -3 and b = 6. In other words, the slope is -3 and the y-intercept is at (0,6).

Plot the *y*-intercept:

(0, 6)

Use the slope to draw the rest of the line:



The slope is $-3 = \frac{-3}{1} = \frac{rise}{run}$

The slope is rewritten as a fraction to reveal the rise (numerator) and run (denominator). The rise is the amount the slope changes on the y-axis and the run is the amount the slope changes on the x-axis.

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Graphs with one variable

If a graph only involves one variable, it can be graphed as a horizontal or vertical line. Below are two examples.



Practice Problems

Graph the following equations.

1) $y = 4x + 1$	6) $y = x$
2) $y = 12x + 3$	7) $3y - 12 = -6x$
3) $y = -3x + 2$	8) $x + 6y = 7$
4) $y = -x + 4$	9) $x = 4$
5) $y = 23x - 2$	10) $y = -2.5$



4.6 Building Lines

You may be asked to state the equation of a line based on the points it passes through or its slope. You may also be asked to write the equation of a line parallel or perpendicular to another line. If you don't know how to graph linear equations, see Section 4.5.

Parallel Lines

Lines are parallel if they have the same slope. Below are two examples of parallel lines. The red lines have different *y*-intercepts than the blue lines, but they have matching slopes. In order to construct parallel lines, just find two lines with the same slope.



Example

Find a line parallel to y = 2x + 3.

Any line that has the same slope (m = 2) is a correct answer! For instance y = 2x + 4 and y = 2x - 24.

Example

Find a line parallel to $y = \frac{5}{7}x$.

Any line that has the same slope $(m = \frac{5}{7})$ is a correct answer! For instance $y = \frac{5}{7}x + 2$ and $y = \frac{5}{7}x - 10$.

Parallel Lines

Lines are perpendicular if they intersect at a 90° angle. Below is an example of a pair of perpendicular lines. The slopes of perpendicular lines are the negative reciprocals of one another.



Example

Find a line perpendicular to y = 2x + 3.

To find the perpendicular slope, take the negative reciprocal of m = 2. The reciprocal of 2 is $\frac{1}{2}$. Then make it negative. Therefore, any equation with the slope $m = -\frac{1}{2}$ is perpendicular to our original equation. For instance, $y = -\frac{1}{2}x + 3$ and $y = -\frac{1}{2}x - 1$.

Example

Find a line perpendicular to $y = -\frac{5}{4}x + 6$.

To find the perpendicular slope, take the negative reciprocal of $m = -\frac{5}{4}$. The reciprocal of $-\frac{5}{4}$ is $-\frac{4}{5}$. Then negate it, causing it to turn positive. (Continued on the next page)

Example Continued

Therefore, any equation with the slope $m = \frac{4}{5}$ is perpendicular to our original equation. For instance, $y = \frac{4}{5}x + \frac{1}{2}$ and $y = \frac{4}{5}x - 3$.

Line Equations

When given characteristics of a line, you can build the corresponding equation. When given a y-intercept and a slope, use the equation y = mx + b. Plug in your slope for m and your y-intercept for b.

When given a point (x_1, y_1) and a slope m, use the equation $y - y_1 = m(x - x_1)$. Then just algebraically rearrange your result into the form y = mx + b.

When given only two points, (x_1, y_1) and (x_2, y_2) , first plug these points into the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find your *m*. Then just use $y - y_1 = m(x - x_1)$ like in the previous case.

Example

Find the equation of the line which passes through the points (1,2) and (4,9).

First we find the slope using $m = \frac{y_2 - y_1}{x_2 - x_1}$. Label the points (x_1, y_1) and (x_2, y_2) . In this case $x_1 = 1$, $y_1 = 2$, $x_2 = 4$, and $y_2 = 9$. So $m = \frac{9-2}{4-1} = \frac{7}{3}$. Now choose either point, and plug it in to $y - y_1 = m(x - x_1)$. We'll use the point (1,2) here. $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{7}{3}(x - 1)$$
$$y - 2 = \frac{7}{3}x - \frac{7}{3}$$
$$y = \frac{7}{3}x - \frac{1}{3}$$

Therefore the equation of the line that passes through the desired points is $y = \frac{7}{3}x - \frac{1}{3}$.

Practice Problems

- 1) Find the equation of a line parallel to y = 3x + 7.
- 2) Find the equation of a line parallel to $y = \frac{1}{2}x 2$.
- 3) Find the equation of a line parallel to $y = -\frac{3}{2}x + 9$.
- 4) Find the equation of a line perpendicular to $y = \frac{1}{2}x + 1$.
- 5) Find the equation of a line perpendicular to y = 5x + 12.
- 6) Find the equation of a line perpendicular to $y = -\frac{2}{7}x 8$.
- 7) Find the equation of the line that has a slope of 2 and intercepts the *y*-axis at y = 7.
- 8) Find the equation of the line that has a slope of $\frac{4}{5}$ and intercepts the *y*-axis at y = -3.
- 9) Find the equation of the line which passes through the points (4,3) and (2,2).
- 10) Find the equation of the line which passes through the points (-4,5) and (0,0).

1)	$y = 3x \pm anything$
2)	$y = \frac{1}{2}x \pm anything$
3)	$y = -\frac{3}{2}x \pm anything$
4)	$y = -2x \pm anything$
5)	$y = -\frac{1}{5}x \pm anything$
6)	$y = \frac{7}{2}x \pm anything$
7)	y = 2x + 7
8)	$y = \frac{4}{5}x - 3$
9)	$y = \frac{1}{2}x + 1$
10)	$y = -\frac{5}{4}x$



Quadratic Equations

5.1 Distribution and Foiling

When two quantities are multiplied, all of the pieces being multiplied must be taken into account. Distribution and foiling are the same thing, technically- but they are defined separately so that each process is easier to understand.

Distribution Example

When multiplying two quantities with no pluses or minuses involved, distribution is simple. Multiply like terms together.

$$5 \cdot (2x) = 5 \cdot 2 \cdot x = 10x$$
$$3x \cdot (4x) = 3 \cdot 4 \cdot x \cdot x = 12x^{2}$$
$$y \cdot (6x) = 6 \cdot x \cdot y = 6xy$$

There are no like terms to combine here, but the multiplication still happens.

Distribution Example

When multiplying two quantities where one has a plus or minus involved, multiply the outer term to each term connected by the plus or minus.



Foiling Example

When multiplying two quantities which both have a plus or minus involved, multiply them in the following way

$$(x+1)(x+2) = x^{2}$$
First
$$(x+1)(x+2) = 2x$$
Outside
$$(x+1)(x+2) = 1x$$
Inside
$$(x+1)(x+2) = 2$$

Last

First, outside, inside, last is where the name Foiling comes from.

Foiling Example

This is the result of the above method.

$$(x + 1)(x + 2) = x^{2} + 2x + 1x + 2 = x^{2} + 3x + 2$$

Thus the distribution results in $x^2 + 3x + 2$.

Practice Problems

Perform the following distributions.

- 1) 2(3*x*)
- 2) 10(10x)
- 3) x(7x)
- 4) $3x(5x^2)$
- 5) x(xy)
- 6) $6yz^{3}(2xy^{3})$
- 7) -x(8x)
- 8) $-3(4x^9)$
- 9) (x+1)(x+2)
- 10) (x+1)(x+5)
- 11) (x+3)(x+3)
- 12) (x+2)(x-9)
- 13) (x-1)(x+4)
- 14) (x+2)(x-2)
- 15) (x+3)(x-3)
- 16) (x+4)(x-4)
- 17) (x-2)(x-2)
- 18) (x-5)(x-6)

1)	6 <i>x</i>	$2(3x) = 2 \cdot 3 \cdot x = 6 \cdot x = 6x$
2)	100 <i>x</i>	$10(10x) = 10 \cdot 10 \cdot x = 100 \cdot x = 100x$
3)	$7x^{2}$	$x(7x) = 7 \cdot x \cdot x = 7 \cdot x^2 = 7x^2$
4)	15 <i>x</i> ³	$3x(5x^2) = 3 \cdot 5 \cdot x \cdot x^2 = 15 \cdot x^3 = 15x^3$
5)	x^2y	$x(xy) = x \cdot x \cdot y = x^2 \cdot y = x^2 y$
6)	$12xy^{4}z^{3}$	$6yz^{3}(2xy^{3}) = 6 \cdot 2 \cdot x \cdot y \cdot y^{3} \cdot z^{3} = 12 \cdot x \cdot y^{4} \cdot z^{3} = 12xy^{4}z^{3}$
7)	$-8x^{2}$	$-x(8x) = -8 \cdot x \cdot x = -8 \cdot x^2 = -8x^2$
8)	$-12x^{9}$	$-3(4x^9) = -3 \cdot 4 \cdot x^9 = -12 \cdot x^9 = -12x^9$
9)	$x^2 + 3x + 2$	$(x+1)(x+2) = x \cdot x + 2 \cdot x + 1 \cdot x + 1 \cdot 2 = x^2 + 2x + 1x + 2 = x^2 + 3x + 2$
10)	$x^2 + 6x + 5$	$(x+1)(x+5) = x \cdot x + 5 \cdot x + 1 \cdot x + 1 \cdot 5 = x^2 + 5x + 1x + 5 = x^2 + 6x + 5$
11)	$x^2 + 6x + 9$	$(x+3)(x+3) = x \cdot x + 3 \cdot x + 3 \cdot x + 3 \cdot 3 = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$
12)	$x^2 - 7x + 18$	$(x+2)(x-9) = x \cdot x - 9 \cdot x + 2 \cdot x - 2 \cdot 9 = x^2 - 9x + 2x + 18 = x^2 - 7x + 18$
13)	$x^2 + 3x - 4$	$(x-1)(x+4) = x \cdot x + 4 \cdot x - 1 \cdot x - 1 \cdot 4 = x^2 + 4x - 1x - 4 = x^2 + 3x - 4$
14)	$x^2 - 4$	$(x+2)(x-2) = x \cdot x - 2 \cdot x + 2 \cdot x - 2 \cdot 2 = x^2 - 2x + 2x - 4 = x^2 - 4$
15)	$x^2 - 9$	$(x+3)(x-3) = x \cdot x - 3 \cdot x + 3 \cdot x - 3 \cdot 3 = x^2 - 3x + 3x - 9 = x^2 - 9$
16)	$x^2 - 16$	$(x+4)(x-4) = x \cdot x - 4 \cdot x + 4 \cdot x - 4 \cdot 4 = x^2 - 4x + 4x - 16 = x^2 - 16$
17)	$x^2 - 4x - 4$	$(x-2)(x-2) = x \cdot x - 2 \cdot x - 2 \cdot x + 2 \cdot 2 = x^2 - 2x - 2x + 4 = x^2 - 4x - 4$
18)	$x^2 - 11x + 30$	$(x-5)(x-6) = x \cdot x - 6 \cdot x - 5 \cdot x + 5 \cdot 6 = x^2 - 6x - 5x + 30 = x^2 - 11x + 30$

5.2 Factoring Expressions

Factoring is the opposite of foiling/distribution. Foiling distributes two quantities into one another, factoring breaks a quantity up into its smaller pieces.

Factoring Example

When factoring an expression, take note what each piece of the expression has in common.

 $5x^{4} + 10x^{3} + 15x^{2}$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$ $= 5 \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$

Now, rewrite the expression using parentheses. On the left side of the parentheses, we will write the numbers and variables each piece has in common. Inside the parentheses we will write the leftovers of each piece.

 $= 5 \cdot x^{2}(x \cdot x + 2 \cdot x + 3)$ $= 5x^{2}(x^{2} + 2x + 3)$

The quadratic leftover in the parentheses cannot be simplified further, so we are done.

In conclusion, the factored form of $5x^4 + 10x^3 + 15x^2$ is $5x(x^2 + 2x + 3)$.

Another factoring example is given on the next page.

Quadratic Factoring Example

When factoring a quadratic, you can guess and check, or you can use the following process. This process starts by filling out an X diagram that looks like this: \bigvee

Consider the quadratic equation $2x^2 - 5x - 3$.

<u>Step 1:</u>

Take the first and last coefficients and multiply them together. Place the result in the top part of the X. Then take the middle coefficient and place it in the bottom part of the X.



<u>Step 2:</u>

Now, find two numbers that multiply together to make the top number, and add together to make the bottom number. In this case, we need two numbers which multiply to make -6 and add to make -5.

The two numbers that meet those conditions are -6 and 1. These numbers are placed in the sides of the X. It doesn't matter which side you put them on.



 $-6 \cdot 1 = -6$ and -6 + 1 = -5

(Continued on next page)

Quadratic Factoring Example Continued

<u>Step 3:</u>

Now that the X diagram is filled out, we use that information to rewrite our quadratic equation as follows.



Notice, the -6 and 1 come from the X diagram.

<u>Step 4:</u>

Our fourth step is to draw a line down the center of the equation and factor each side separately.

Factor this side.

$$2x^{2} - 6x + 1x - 3$$

$$2x^{2} - 6x$$

$$1x - 3$$
Factor this side.

$$2x(x - 3)$$

$$1(x - 3)$$
Factor this side.

<u>Step 5:</u>

Our final step is to write our factors! One of the factors will be the expression inside the parentheses. Notice how they are both the same! So one of our factors is (x - 3). The other factor is made up of the remaining pieces. In this case, the 2x and the positive 1. So our other factor is (2x + 1).

In conclusion, the factored form of $2x^2 - 5x - 3$ is (x - 3)(2x + 1).

Practice Problems

Factor the following expressions.

1) 3x - 92) $5x^2 + 2x$ 3) $2x^2 + 4x$ 4) $10x^3 + 30x^2$ 5) $x^2 + 3x + 2$ 6) $x^2 + 5x + 6$ 7) $x^2 + 6x + 5$ 8) $x^2 + 8x + 16$ 9) $x^2 + 6x + 9$ 10) $x^2 + 11x + 30$ 11) $x^2 - x - 2$ 12) $x^2 - 8x + 12$ 13) $x^2 - 13x - 30$ 14) $5x^2 - 9x - 2$ 15) $6x^2 + 13x + 6$ 16) $x^2 - 4$ 17) $x^2 - 9$ 18) $x^2 - 1$ 19) $x^2 - 16$

Hint: When you see quadratics in this form, you can rewrite them with +0xin the middle.

 $x^2 - 4 = x^2 + 0x - 4$

1)	3(x-3)
2)	x(5x + 2)
3)	2x(x+2)
4)	$10x^2(x+3)$
5)	(x+1)(x+2)
6)	(x+2)(x+3)
7)	(x+1)(x+5)
8)	(x + 4)(x + 4)
9)	(x+3)(x+3)
10)	(x+5)(x+6)
11)	(x+1)(x-2)
12)	(x-2)(x-6)
13)	(x-10)(x-3)
14)	(5x+1)(x-2)
15)	(3x+2)(2x+3)
16)	(x+2)(x-2)
17)	(x+3)(x-3)
18)	(x+1)(x-1)
19)	(x+4)(x-4)

5.3 Solving Quadratic Equations

Solving quadratic equations requires factoring.

Example

First, use algebra to rearrange the quadratic equation so that it is equal to zero.

$$x^2 = -3x - 2$$
$$x^2 + 3x + 2 = 0$$

Next, factor the left side.

$$x^{2} + 3x + 2 = 0$$
$$(x + 2)(x + 1) = 0$$

Now, set each of those expressions equal to zero and solve.

x + 2 = 0 x + 1 = 0x = -2 x = -1

The solutions to the quadratic equation are x = -2 and x = -1. Try plugging x = -2 into the original equation. Then try plugging x = -1 into the original equation. Note you will get 0 = 0 each time, meaning both values for x satisfy the equation!

Practice Problems

Solve the following equations. (If you have already completed the factoring practice problems in Section 5.2, you can use that work to focus on the solving process here).

- 1) $x^2 + 5x + 6 = 0$
- 2) $x^2 + 8x + 16 = 0$
- 3) $x^2 x 2 = 0$
- 4) $x^2 8x = -12$
- 5) $x^2 = 4$
- 6) $5x^2 = 9x + 2$
- 7) $-x^2 = 6x + 9$
Answers

1)	x = -2, x = -3	$x^{2} + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0 \rightarrow x + 2 = 0$ and $x + 3 = 0$
2)	x = -4	$x^{2} + 8x + 16 = 0 \rightarrow (x + 4)(x + 4) = 0 \rightarrow x + 4 = 0$ and $x + 4 = 0$
3)	x = -1, x = 2	$x^{2} - x - 2 = 0 \rightarrow (x + 1)(x - 2) = 0 \rightarrow x + 1 = 0 \text{ and } x - 2 = 0$
4)	x = 2, x = 6	$x^{2} - 8x + 12 = 0 \rightarrow (x - 2)(x - 6) = 0 \rightarrow x - 2 = 0$ and $x - 6 = 0$
5)	x = -2, x = 2	$x^{2} - 4 = 0 \rightarrow (x + 2)(x - 2) = 0 \rightarrow x + 2 = 0$ and $x - 2 = 0$
6)	$x = -\frac{1}{5}, x = 2$	$5x^2 - 9x - 2 = 0 \rightarrow (5x + 1)(x - 2) = 0 \rightarrow 5x + 1 = 0 \text{ and } x - 2 = 0$
7)	x = -3	$x^{2} + 6x + 9 = 0 \rightarrow (x + 3)(x + 3) = 0 \rightarrow x + 3 = 0$ and $x + 3 = 0$



Sets and Probability

6.1 Sets

Sets are collections of objects. They are denoted in a list between brackets $\{ \}$. For these exercises, the sets will only contain numbers. The numbers/objects in a set are called **elements** of the set. If a set has no elements, it is called **empty** and is denoted $\{ \}$ or \emptyset .

Notation

The set A contains the numbers 1, 2, 3, and 4:	$A = \{1, 2, 3, 4\}$
The set B contains the numbers 3, 4, 5, and 6:	$B = \{3, 4, 5, 6\}$

Rules

1) The order the numbers are listed does not matter.

$$\{1, 2, 3\} = \{2, 3, 1\}$$

2) Numbers appearing more than once should be rewritten as just one number.

$$\{1, 2, 2, 3\} = \{1, 2, 3\}$$

3) Remember order of operations! Perform operations in parentheses *first*!

Definition

The **union** of two sets A and B, denoted $A \cup B$, is the set of all elements contained by A, B, or both. The example below in the right column shows the union of the sets in the left column.

	The Union of A and B
$A = \{1, 2, 3, 4\}$	
$B = \{3, 4, 5, 6\}$	$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Definition

The **intersection** of two sets A and B, denoted $A \cap B$, is the set of all elements that are members of *both* A and B. The example below in the right column shows the intersection of the sets in the left column.

	The Intersection of A and B
$A = \{1, 2, 3, 4\}$ B = {3, 4, 5, 6}	$A \cap B = \{3, 4\}$

Visualization



Order of Operations

Evaluate set operations in parentheses first. To evaluate $A \cap (B \cup C)$, find the union of B and C first (B \cup C), and then intersect the resulting set with A (A \cap (B \cup C)).



Practice Problems

Consider the following sets.

$A = \{1, 2, 3, 4, 5\}$	$D = \{10, 20, 30, 40, 50\}$
$\mathbf{B} = \{2, 4, 6, 8, 10\}$	$E = \{2, 3, 5, 6, 7, 11\}$
$C = \{1, 3, 5, 7, 9\}$	$\mathbf{F} = \mathbf{\emptyset}$

Evaluate the following.

1) BUC	7) B∩C	13) A∩(B∪C)
2) AUB	8) D∩B	14) E∪(D∩B)
3) AUC	9) A∩F	15) (AUE)UD
4) A∩B	10) AUF	16) (B∩A)∪E
5) B∩D	11) DUC	17) A∪(F∩C)
6) A∩E	12) A∩A	18) $A \cap (B \cap C)$

Answers

1)	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
2)	{1, 2, 3, 4, 5, 6, 8, 10}
3)	{1, 2, 3, 4, 5, 7, 9}
4)	{2, 4}
5)	{10}
6)	{2, 3, 5}
7)	Ø
8)	{10}
9)	Ø
10)	{1, 2, 3, 4, 5}
11)	{1, 3, 5, 7, 9, 10, 20, 30, 40, 50}
12)	{1, 2, 3, 4, 5}
13)	{1, 2, 3, 4, 5}
14)	{2, 3, 5, 6, 7, 10, 11}
15)	$\{1, 2, 3, 4, 5, 6, 7, 10, 11, 20, 30, 40, 50\}$
16)	$\{2, 3, 4, 5, 6, 7, 11\}$
17)	$\{1, 2, 3, 4, 5\}$
18)	Ø

6.2 Probability

Probability is the likelihood that an event will occur.

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

The probability of an event ranges between 0 and 1 (0 is impossible; 1 is certain). It can also be represented as a percentage (0% to 100%).

Notation

P(A) means "Probability of Event A"

Rules

- If the probability of A is impossible, P(A) = 0.
- If the probability of A is certain, P(A) = 1.
- In all cases, $0 \le P(E) \le 1$
- If two or more events are mutually exclusive and constitute all the outcomes, the sum of their probability is 1.
- The probability that an event E will not occur is 1 P(A)
- For independent events A and B: $P(A \text{ and } B) = P(A) \cdot P(B)$
- When two events, A and B, are mutually exclusive, the probability that A or
 B: P(A or B) = P(A) + P(B)
- When two events, A and B, are mutually inclusive, the probability that A or
 B: P(A or B) = P(A) + P(B) P(A and B)
- If events A and B are not independent, then the probability that both events occur: P(A and B) = P(A) · P(B|A), P(B|A) is the notation for the probability of B given A. Or,

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Terminology

Simple Probability: Finding the probability of a single event happening.

Mutually Exclusive: Two things are mutually exclusive if they cannot occur at the same time.

Simple Probability Example

If a coin is tossed, what is the probability of getting heads?

Solution:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$ Number of ways head can happen = 1 Total number of outcomes = 2 Therefore,

$$P(heads) = \frac{1}{2}$$

Simple Probability Example

Given a standard six-sided die, determine the probability of rolling a five.

<u>Solution:</u>

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

Number of ways it can happen = 1

There is only one side with a 5, so there is 1 chance that a die will land on 5.

Total number of outcomes = 6

When a single die is thrown, there are six possible outcomes: 1, 2, 3, 4, 5, 6. Therefore,

$$P(5)=\frac{1}{6}$$

Simple Probability Example

A bag contains 4 white balls, 6 black balls, and 1 green ball. If you randomly pick a ball, what is the probability of getting:

a. a green ball?b. a black ball?

Solution for getting a green ball:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

Number of ways it can happen = 1

Total number of outcomes = 4+6+1 = 11

Therefore,

$$P(green) = \frac{1}{11}$$

Solution for getting a black ball:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

Number of ways it can happen = 6 Total number of outcomes = 11 Therefore,

$$P(black) = \frac{6}{11}$$

Simple Probability Example

A deck of cards contains 52 cards, 13 from each suit. If a card from the deck is flipped over, what is the probability that the card is an Ace of Spades? What is the probability that it is a Spade?

Solution for selecting an Ace of Spades:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

Number of ways it can happen = 1

Total number of outcomes = 52

Therefore,

$$P(Ace \ of \ Spades) = \frac{1}{52}$$

Solution for selecting a Spade in general:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

Number of ways it can happen = 13 (Because there are 13 spades) *Total number of outcomes = 52*

Therefore,

$$P(Ace of Spades) = \frac{13}{52} = \frac{1}{4}$$

Simple Probability Example

If there is a 20% chance of rain on Saturday, what is the probability there will be no rain on Saturday?

Solution:

We know if two or more events are mutually exclusive and constitute all the outcomes, the sum of their probability is 1. i.e.

 $P(A) + P(\overline{A}) = 1$

Therefore,

P(rain) + P(not rain) = 1

We know,

$$P(rain) = 20\%$$

= $\frac{20}{100} = \frac{1}{5}$ Substitute probability
and solve

Therefore,

$$\frac{1}{5} + P (not rain) = 1$$

$$P (not rain) = 1 - \frac{1}{5} = \frac{4}{5} or 80 \%$$

Terminology

Compound Probability: The probability of the joint occurrence of two or more simple events. Either all events can be true at the same time or one of them can be true at a time. If all the events need to be true at the same time, we multiply the probabilities. If one or more parts of the event holds true, we add the probabilities. In the case of duplication, we need to subtract the duplicated probability.

• Probability of A and Probability of B:

$$P(A \text{ and } B) = P(A) * P(B)$$

• Probability of A or Probability of B (mutually exclusive):

$$P(A \text{ or } B) = P(A) + P(B)$$

• Probability of A or Probability of B (inclusive):

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Compound Probability Example

If you flip a coin three times, what is the probability that you will get heads for all three flips?

Solution:

$$Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$$

For each coin, there is one head and two possibilities.

Number of ways it can happen = 1

Total number of outcomes = 2

Therefore, for each coin, $P(head) = \frac{1}{2}$

Since we have three coin flips and they are independent of each other, we multiply.

 $P(head and head and head) = P(head) \cdot P(head) \cdot P(head)$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$
$$= \frac{1}{8}$$

Compound Probability Example

If you roll two fair six-sided dice, what is the probability of getting a 2 on both dice? What is the probability that you do not get 2 on both dice?

Solution for getting a 2 on both dice:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$ For each die, *Number of ways an individual die can land on 2 = 1 Total number of outcomes = 6* (Continued on next page)

Compound Probability Example Continued

Since there are two dice and they are independent of each other, we multiply.

$$P(2 \text{ and } 2) = P(2) * P(2)$$
$$= \frac{1}{6} * \frac{1}{6}$$
$$= \frac{1}{36}$$

Solution for not getting a 2 on both dice:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

For each die,

Number of ways the die could not land on 2 = 5

Total number of outcomes = 6

Since there are two dice and they are independent of each other, we multiply.

P(2 and 2) = P(not 5) * P(not 5) $= \frac{5}{6} * \frac{5}{6}$ $= \frac{25}{36}$

Compound Probability Example

A bag contains 6 red candies, 4 green candies, and 4 blue candies.

If we take one candy out of the bag, followed by another candy without putting the first one back in the bag, what is the probability that the first candy will be green and the second will be red?

Solution:

 $Probablity of an event = \frac{\# of ways it can happen}{total number of outcomes}$

(Continued on next page)

Compound Probability Example Continued





For the first candy,

Number of ways to pick green = 4

Total number of outcomes = 6 + 4 + 4 = 14

Therefore,

$$P(green) = \frac{4}{14} = \frac{2}{7}$$

For the second candy,

Number of ways to pick red = 6

Total number of outcomes = 6 + 3 + 4 = 13 (As we already picked a green candy)

Therefore,

$$P(red) = \frac{6}{13}$$

Since they are independent of each other, we multiply,

Probability (green and red) = P(green) * P(red)

$$=\frac{2}{7} * \frac{6}{13}$$
$$=\frac{12}{91}$$

Compound Probability Example

A single six-sided die is rolled. What is the probability of rolling a 2 or a 5?

Solution:

$$P(2) = \frac{1}{6}$$

$$P(5) = \frac{1}{6}$$

$$P(2 \text{ or } 5) = P(2) + P(5)$$

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$= \frac{1}{3}$$

Compound Probability Example

A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a king or a club?

Solution:

Number of ways a king can be chosen = 4 (There are 4 kings)

Number of ways a club can be chosen = 13 (There are 13 clubs)

However, we counted the king of club twice. So we need to subtract it.

Number of ways the king of clubs can be chosen = 1 (one king of club)

Total number of outcomes = 52

Therefore,

$$P(king \text{ or } club) = P(king) + P(club) - P(king \text{ of } clubs)$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$=\frac{16}{52}=\frac{4}{13}$$

Compound Probability Example

The probability of a person having a car accident is 0.09. The probability of a person driving while intoxicated is 0.32 and probability of a person having a car accident while intoxicated is 0.15. What is the probability of a person driving while intoxicated or having a car accident?

```
Solution:

P(intoxicated or accident)

= P(intoxicated) + P(accident) - P(intoxicated and accident)

= 0.32 + 0.09 - 0.15

= 0.26
```

Terminology

Conditional Probability: The conditional probability of an event B is the probability that the event will occur given the knowledge that an event A has already occurred.

If events A and B are dependent, then the probability that both events occur is defined by

$$P(A \text{ and } B) = P(A) * P(B|A)$$

or

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Conditional Probability Example

At a school, 60% of the athletes play baseball, and 24% of the athletes play baseball and football. What percent of those that play baseball also play football?

Solution: P(baseball) = 60% = 0.60 P(baseball and football) = 24% = 0.24 Probability of players playing football given the probability of baseball = ? $P(football | baseball) = \frac{P(baseball and football)}{P(baseball)}$ $= \frac{0.24}{0.6} = 0.4$ = 40%

Conditional Probability Example

The table below shows a survey of 50 registered voters in a city. Each voter was asked whether they planned to vote "yes" or "no" on two different issues. If a voter who plans to vote "yes" on issue P is randomly selected, what is the probability that voter also plans to vote "yes" on issue Q?

	Plans to vote "Yes" on issue Q	Plans to vote "No" on issue Q	Total
Plans to vote	8	12	20
"Yes" on issue P			
Plans to vote "No"	14	16	30
on issue P			
Total	22	28	50

(Solution on next page)

Conditional Probability Example Continued

Solution:

There are 8 voters who plan to vote "Yes" on both issues (P and Q)

There are 20 voters who plan to vote "Yes" on issue P.

Probability of voters to vote "Yes" on issue Q given that they plan to vote "Yes" on issue P = ?

$$P(Q|P) = \frac{P(P \text{ and } Q)}{P(P)}$$
$$= \frac{8}{20} = \frac{2}{5}$$

Conditional Probability Example

A math teacher gave two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

Solution:

$$P(second | first) = \frac{P(first and second)}{P(first)}$$
$$= \frac{0.25}{0.42} = 0.60 = 60\%$$

Practice Problems

- 1) If a dice is rolled once. What is the probability that it will show an even number?
- 2) A day of the week is chosen at random. What is the probability of choosing a Saturday or Sunday?
- 3) If you flip a coin and roll a die at the same time, what is the probability you will flip a head and roll a five?
- 4) A number from 1 to 10 is chosen at random. What is the probability of choosing an even number?
- 5) A bag contains 4 red marbles and 4 blue marbles. Two marbles are drawn at random without replacement. If the first marble drawn is blue, what is the probability the second marble is also blue?
- 6) A box contains 5 green pens and 7 yellow pens. Two pens are chosen at random from the box without replacement. What is the probability they are both yellow?
- 7) If you roll two fair six-sided dice, what is the probability that the sum is 9 or higher?
- 8) Two cards are chosen at random without replacement from a pack of a standard playing cards. If the first card chosen is an Ace, what is the probability the second card chosen is a King.
- 9) In a city, 48% of all teenagers own a skateboard and 39% of all teenagers own a skateboard and roller blades. What is the probability that a teenager owns roller blades given that the teenager owns a skateboard?
- 10) From the following probability table, what is the probability a randomly selected person is a hiker, given that they own a dog?

	Have dog	Do not have dog	Total
Hiker	0.41	0.18	0.59
Non-Hiker	0.35	0.06	0.41
Total	0.76	0.24	1

Answers

1)	$\frac{1}{2}$
2)	$\frac{2}{7}$
3)	$\frac{1}{12}$
4)	$\frac{1}{2}$
5)	$\frac{3}{7}$
6)	$\frac{7}{22}$
7)	$\frac{5}{18}$
8)	<u>4</u> 51
9)	81.25%
10)	0.477



Descriptive Statistics

7.1 Descriptive Statistics

Descriptive statistics are used to describe or summarize data in a meaningful way. Mean, median, mode, range, minimum, and maximum are ways to summarize sample data.

Terminology

The **mean** is an average of the data set. Mean is calculated by finding the sum of the observations divided by the number of observations.

Example

Find the mean of the data set: 100, 98, 105, 90, 102

There are 5 numbers in the data set:



Example

The mean of four numbers is 71. If three of the numbers are 58, 75, and 89, what is the value of the fourth number?

mean = 71# of observations = 4
Let the missing number be x. sum = 58 + 75 + 89 + x = 222 + x $mean = \frac{sum}{\# of observations}$ $71 = \frac{222 + x}{4}$ 71 * 4 = 222 + x x = 71 * 4 - 222 x = 285 - 222 = 62

Therefore, the fourth number is 62.

Terminology

The **median** is the middle value in a set of data. It is calculated by first listing all of the data values in numeric order and then locating the value that is in the middle of the list.

If the amount of data values is odd, there will be one middle value- which is the median. If the amount of data values is even, there will be two middle values, and therefore, we need to calculate the average of the two middle values to get the median.

Example

Find the median of the data set. 13, 18, 13, 14, 13, 16, 14, 21, 13

Let's rewrite the data in numeric order.

13	13	13	13	14	14	16	18	21
1	2	3	4	5	6	7	8	9

There are 9 numbers.

middle value =
$$\frac{9+1}{2} = 5$$

Therefore, median = 14

Example

A student achieved the following scores on 10 math quizzes.

68, 50, 70, 62, 71, 58, 81, 85, 63, 79

What is the median score achieved by the student?

Let's rewrite the data in numeric order.



Since there is an even number of scores in the data set, we calculate the median by taking the mean of the two middlemost numbers.

$$median = \frac{68 + 70}{2} = 69$$

Terminology

The **mode** is the number that appears most frequently in the set of data. If there is no repetition of any number, then there is no mode. However, if there is more than one most frequently repeated number, then there will be more than one mode.

Example

Find the mode of the data set. 13, 18, 13, 14, 13, 16, 14, 21, 13

Let's count the repetitions of the numbers.

Data	Repetition
13	4
14	2
16	1
18	1
21	1

Since the most frequently appearing data is 13, mode = 13

Example

Find the mode of the data set. 2, 5, 2, 3, 5, 4, 7

Let's count the repetition of the numbers.

2	2
3	1
4	1
5	2
7	1

Since the most frequently appearing data is 2 and 5, mode = 2 and 5

Terminology

The **range** is difference between smallest and largest numbers in the data set. The minimum number is the smallest value in the data set. The maximum number is the largest value in the data set.

Example

Find the range of the data set: 13, 18, 13, 14, 13, 16, 14, 21, 13

When we observe the date, 13 is the smallest number and 21 is the largest number.

$$smallest = 13$$

 $largest = 21$

Therefore,

range = largest - smallest= 21 - 13= 8

Example

In a certain match, following five wrestlers were playing. What was the range of weight among the wrestlers.

Wrestlers	Weight in pounds
Big Cass	276
Bull Buchanan	275
Donovan Dijak	270
Wade Barrett	246
Giant Gonzales	460

$$smallest = 246$$
$$largest = 460$$
$$range = largest - smallest$$
$$= 460 - 246$$
$$= 214$$

Graphical Representation

Terminology

A histogram is a bar graph where the data is represented in equal intervals.

Example

In the following graph, how many students got the highest score?



The highest score ranges from 80 to 100. Based on the histogram chart, there were about 15 students. Therefore, the answer is 15.



Example

The histogram below shows the height (in cm) distribution of 30 people. How many people have heights between 160 and 170 cm? How many people have heights less than 160 cm?





Solution for heights between 160 and 170 cm:



We need to look at the third bar because it ranges from 160 cm to 170 cm. The bar indicates that there are 7 people.

Therefore, the answer is 7.

(Continued on next page)

Example Continued

Solution of number of people less than 160 cm:



For the people less than 160 cm height, we have to look at the bars from two categories – 140 cm to 150 cm and 150 cm to 160 cm.

Therefore,

140-150: 6 people

150-160: 9 people

Total people less than 160 cm is 6 + 9 = 15 people

Example

How many students got a grade of 'A' based on the following chart.



(Continued on next page)

Example Continued



When we observe the bar A, the bar lines up with 5. Therefore, the answer is 5 students.

Example

Logan sells four different flavors of jam at an annual farmers market. The graph below shows the number of jars of each type of jam they sold at the market during the first two years. Which flavor of jam had the greatest increase in number of jars sold from Year 1 to Year 2?



Source: Adapted from CollegeBoard

For this problem, we have to calculate the difference between year 1 and year 2 of all the jars.

Jars	Year 1	Year 2	Difference (Year 2 – Year 1)			
Blueberry	10	18	8			
Grape	25	30	5			
Peach	8	12	4			
Strawberry	35	20	-15			

Although strawberry has the highest difference, it is not the answer because the number of jars decreased. Blueberry jars increased from 10 to 18 with an increase of 8 jars. Therefore, blueberry is the answer.

Terminology

A scatter plot is a graph which represents a set of points on the x- y axes.

Example

A sample of 5 students have the following body weights and heights. The data is represented in a scatter plot graph. Based on the graph, if a student has a weight of 44 kg, how tall is the student?

Height in cm	110	114	120	125	132
Weight in kg	35	38	44	49	56

(Graph and answer on next page)



As previously stated, the points are x-y points of the given data. The point (110, 35) is the first point, and the (132, 56) is the last point.



Example

A research on two stock companies reveals that the closing prices of stocks were positively correlated to each other. The following chart shows the stock prices of the companies and a line of best fit. Based on the chart, if the price of stock A is \$15, what would the price of stock B be?


Terminology

A range bar graph represents a range of data for each independent variable.

Example





Temperature of Warmest Months



In June, temperatures range from 50° F to 70° F.

In July, temperatures range from 52° F to 90° F.

In August, temperatures range from 50° F to 95° F, 95 being the highest.

In September, temperatures range from 40° F to 70° F.

Therefore, the warmest month is August. The highest temperature is 95° F.

Example

The box plot below summarizes the resting heart rates, in beats per minute, of the members at a gym. Which of the following could be the range of resting heart rates, in beats per minute?



Adapted from Khan Academy

The chart shows resting heart rates (beats/minute), and we are asked to calculate range of resting heart rates.

We know *range* = *maximum* – *minimum*.

Resting Heart Rates (beats per minute)



Based on the chart we can find maximum and minimum values.

$$maximum = 101$$
$$minimum = 39$$
$$range = 101 - 39$$
$$= 62$$

Therefore, the answer is 62 beats per minute.

Practice Problems

- 1) Find the mean, median, mode, and range of 8, 5, 7, 10, 15, 21.
- 2) Find the mean, median, mode, and range for the set of numbers 3, 3, 9, 14, 6.
- 3) The following graph represents age distribution of 10 students in an high school class. Find the mean, median, mode, and range of the values:



4) The following table shows the number of rainy days in five cities in Colorado last summer. If the mean of the data set is 6, find the number of rainy days in City 5.

Cities	City 1	City 2	City 3	City 4	City 5
Number or rainy days	5	7	2	9	?

5) If the mean of the data set shown on the graph to the right is 4 bubbles, find the number of bubbles Hannah blew.



Adapted from Khan Academy

- 6) 20 bricks have a mean weight of 24 pounds, and 30 similar bricks have a weight of 23 pounds each. What is the total weight of all the bricks?
- 7) A local high school developed a bar diagram (shown below) for the memberships on each of the 4 clubs. How many more members does the Music club have than the Science club?



8) A health care expert studied worldwide healthcare spending over time. In 2005, how much more did Bhutan spend on healthcare per capita than Senegal? (Graph to the right)



Healthcare Spending per Capita

9) A new clinic presented the total number of patients based on certain age groups. How many patients are registered in the clinic whose age is less than 20?



10) Sam went swimming and swam laps for the last 7 days. How many laps did they make in average? Round to the nearest decimal if necessary.



Answers

1)	Mean = 11, Median = 9, Mode = No mode, Range = 16
2)	Mean = 7, Median = 6, Mode = 3, Range = 11
3)	Mean = 10.6, Median = 9.5, Mode = 9, Range = 12
4)	7
5)	1
6)	1170 pounds
7)	30
8)	\$10
9)	9
10)	11



Geometry Concepts

8.1 Two-Dimensional Shapes

Geometry word problems consist of relating two things: the information you are given and your knowledge of formulas. Often you will be asked to recall a formula from memory (such as area or perimeter) and use that formula to solve the problem. The following is a list of common 2D shapes and their formulas. You are more likely to see squares, rectangles, and circles on the test.

Terminology

Perimeter: The total distance around the edge of a shape. Finding the perimeter means adding up the lengths of all its sides.

Area: The size of a surface.

Hypotenuse: The longest side of a right triangle.

Rectangles

Perimeter:	P = L + L + W + W	
	= 2L + 2W	
Area:	$A = L \cdot W$	



Squares

Perimeter: P = x + x + x + x= 4x Area: $A = x \cdot x$ = x^2



Triangles

Perimeter: P = A + B + C**Area:** $A = \frac{1}{2} \cdot Base \cdot Height$

$$=\frac{1}{2}B\cdot H$$



Right Triangles

Pythagorean Theorem:
$$c^2 = a^2 + b^2$$



Circles

Circumference: $C = 2\pi R$ **Area:** $A = \pi R^2$ **Diameter:** D = 2R



Practice Problems

Find the area of the given shaded region. If more information than the diagram is given, use that information to solve the problem.



4) If the length of this rectangle is 4 times the width, and the area is 36 units squared, what is the width?



5) If the hypotenuse of the triangle below is $\sqrt{2}$, what is the area of the triangle?



6) The area of the figure below is 144 units squared. What is the value of x?



7) If z = 2, find the Area and the Perimeter.



8) Given that the circumference of the circle is 8π , what is the area of the shaded region?



Answers

1)	$A = 12x^2$
2)	$A = 4t^2$
3)	$A = \pi x^2$
4)	w = 3
5)	$A = \frac{1}{2}$
6)	x = 4
7)	$P = 28, \qquad A = 24$
8)	$A = 64 - 16\pi$

8.2 Three-Dimensional Shapes

The following is a list of common 3D shapes and their formulas. You are more likely to see cubes and spheres on the test.

Terminology

Surface Area: The total area of the surfaces of a three-dimensional object.

Volume: The amount of space inside of a solid figure.

Rectangular Prisms

Surface Area: SA = 2(LW + LH + HW)

Volume: $V = L \cdot W \cdot H$



Cubes

Surface Area: $SA = 6x^2$

Volume: $V = x^3$



Triangular Prisms

In prisms there are two triangles (front and back), two surfaces (right and left), and one base surface (bottom).

Surface Area: $SA = 2 \cdot \frac{1}{2}BH + 2LW + BL$ = BH + 2LW + BLVolume: $V = Area of Triangle \cdot Length$

$$=\frac{1}{2}B\cdot H\cdot L$$



Spheres

Surface Area: $SA = 4\pi R^2$ Volume: $V = \frac{4}{3}\pi R^3$



Practice Problems

Find the surface area and volume of the following shapes. Use the information and diagram provided.

1) The length of the cube is 5 cm.



2) Given a smaller cube of a Rubik cube has a length of 2 cm.



3) The following is a prism.



4) The soccer ball has a diameter of 9 inches.



Answers

- 1) $SA = 150 \ cm^2, V = 125 \ cm^3$
- 2) $SA = 216 \ cm^2, V = 216 \ cm^3$
- 3) $SA = 240 \ cm^2, V = 144 \ cm^3$
- 4) $SA = 81\pi \ cm^2, V = 121.5\pi \ cm^3$



Word Problems

9.1 Setting up an equation

An important skill needed on the test is translating sentences into equations.

In this section you will practice translating from a phrase into a mathematical expression. Sometimes this can be straightforward and sometimes it can be very challenging. There is no single perfect strategy to translate, so as you go forward think of these questions:

-Can you explain why the answer you chose is correct?

-If you plugged a number into the expression you chose, would that expression manipulate the number in the way you expect it to?

-Are there any multiple choice answers you can eliminate right away? (Ones that don't make sense or are trying to mislead you).

Addition (+)

When trying to determine if the + operation should be used in an expression, look for key words or phrases like 'sum', 'plus', 'total', 'and', 'together', 'additional', 'added to', 'combined with', 'more than', 'increased by'.

Subtraction (-)

When trying to determine if the – operation should be used in an expression, look for key words or phrases like 'minus', 'reduce', 'difference', 'left', 'change', 'less', 'fewer', 'decreased by', 'less than', 'take away', 'subtract from', 'how much less', 'how much left'.

Multiplication (×)

When trying to determine if the \times operation should be used in an expression, look for key words or phrases like 'product', 'times', 'twice', 'doubled', 'tripled', 'per', 'squared', 'cubed', 'of', 'multiplied by'.

Division (÷)

When trying to determine if the \div operation should be used in an expression, look for key words or phrases like 'ratio', 'quotient', 'average', 'over', 'into', 'half of', 'a third of', 'split evenly', 'shared equally', 'divided by'.

Practice Problems

- Which of the following expressions is twice as much as x?
 - A) x + 2B) x^2 C) x * xD) 2x
- 2) Which of the following expressions is 3 times as much as the sum of y and z?
 - A) 3 * y + zB) 3 + y + zC) y + z * 3D) (y + z) * 3
- 3) Which of the following expressions is one less than the sum of x and y?

A)
$$(x + y) - 1$$

B) $x * y - 1$
C) $(x - y) + 1$
D) $-1 + (x - y)$

- 4) Which expression represents 10 more than twice the value of x?
 - A) 2 + x + 10B) 2x + 10C) x + 20D) 10x + 2

- 5) Which expression represents y minus the sum of x and z?
 - A) y x + zB) y + x + zC) y(x - z)D) x - z + y
- 6) What is 2 more than the product of x and y?
 - A) (x + 2) * yB) 2 + x + yC) 2 + (x * y)D) (y + 2) * x
- 7) Which expression represents the difference between x and y, multiplied by z?
 - A) x + y + zB) (x - y) * zC) z * (x + y)D) x - z - y
- 8) Which expression represents three times the quotient of x and y?

A)
$$3 * \left(\frac{y}{x}\right)$$

B) $3xy$
C) $3 * x^2y^2$
D) $3 * \left(\frac{x}{y}\right)$

Answers

1)	D
2)	D
3)	A
4)	В
5)	A
6)	C
7)	В
8)	D

9.2 Word Problem Practice

The following is word problem practice. A majority of the test will be interpreting a situation and using your math skills to solve for something.

When you approach word problems, keep these guidelines in mind:

-What information is a distraction (unneeded)?

-What information is given? Can you list it out or should it be written as an equation?

-What does the word problem want you to solve for in the end? (Keep this in mind as you proceed).

Practice Problems

Solve the following word problems.

1) Alex the farmer wants to fence off a rectangular area of 100 square feet on their farm and wants the side closest to their house to be 25 feet long. How many feet of fencing do they need to purchase to complete the project?

A) 29 feet	C) 100 feet
B) 58 feet	D) 75 feet

2) Erin makes $\frac{x}{y}$ dollars selling lemonade, Glenn makes $\frac{x-y}{y}$ dollars as a cook, and James only makes $\frac{2}{3}$ as much as what Glenn makes. Which of the following equations represents the total combined income of all three people?

A)
$$\frac{2x-2y}{3y}$$

B) $1 + \frac{2x-2y}{3y}$
C) $1 + \frac{6x-3y}{3y}$
D) $\frac{8x-5y}{3y}$

3) If Lee can walk two miles in one hour, how many miles could they walk in four and a half hours?

A) 8 ¼ miles	C) 9 ¼ miles
B) 8 ½ miles	D) 9 miles

4) If the price of an item is represented by *k*, how much is the item worth if it is discounted by 23%?

A)
$$0.23k$$
C) $k - 0.23$ B) $0.77k$ D) $k + 0.77k$

5) Micah drops a ball from a cliff that is 200 meters high. If the distance of the ball down the cliff is represented by $D = \frac{1}{2}gt^2$, how long will it take the ball to travel 122.5 meters down the cliff? (Assume $g = 9.8 \frac{m}{s^2}$)

A) 5 seconds	C) 12 seconds
B) 10 seconds	D) 15 seconds

6) The area of a right triangle is given by the equation $A = \frac{1}{2}bh$, where *b* represents the base and *h* represents the height. If a right triangle has an area of 10 and a height of 5, what is the length of its base?

A) 5	C) 4
B) 2	D) 25

7) In the beginning of the month, Morgan owes Payton, John, and Susan
 \$3.50 each. If Morgan receives a paycheck of \$250.00 and repays Payton, John, and Susan, how much will she have left afterwards?

A) \$239.50	C) \$246.00
B) \$241.50	D) \$243.50

8) Taylor has \$90 in their checking account. They make purchases of \$22 and \$54. Then they overdraft their account with a purchase of \$37 resulting in a \$35 fee. The next day, Taylor deposits a check for \$185. What is the balance of their bank account after the deposit?

A) - \$58.00	C) -\$127.00
B) \$58.00	D) \$127.00

9) The temperatures on a certain planet can be as high as 102° C during the day and as low as -35° C at night. How many degrees does the temperature drop from day to night?

A) 67°C	C) 137°C
B) 102°C	D) 35°C

10) To calculate the Body Mass Index of an individual, their weight (in kilograms) is divided by the square of their height (in meters). If Mike weighs 108 kilograms and stands 2 meters tall, what is his BMI?

A) 27	C) 22
B) 24	D) 18

11) Jasmine buys a car at a major dealership. She pays a single payment of \$5,775 for it. If 5% of that payment was taxes and fees, what was the actual price of the car? (Round to the nearest dollar)

A) \$6,064	C) \$3,850
B) \$5,500	D) \$5,486

12) The perimeter of a rectangle is three times the length. The length is six inches longer than the width. What are the dimensions of this rectangle?

A) 12 in × 6 in
B) 15 in × 9 in
C) 18 in × 10 in
D) 10 in × 4 in

Answers

1)	В
2)	D
3)	D
4)	В
5)	Α
6)	C
7)	Α
8)	D
9)	C
10)	A
11)	D
12)	Α



10.1 Practice Test 1

Answer the following questions.

- 1) Which of the following expressions is twice as much as the sum of x and y?
 - A) 2x + y
 - B) x + 2y
 - C) 2(x + y)
 - D) 2(x y)
- 2) Solve the equation for x: 7x + 3x = 10(x 2) + 5x
 - A) -4B) $-\frac{2}{5}$ C) 4 D) 5x
- 3) How many kilometers are in 6 cm? (100 cm = 1 m and 1000 m = 1 km)
 - A) .00006 km
 - B) .0006 km
 - C) .006 km
 - D) 6 *km*

- 4) Robert sells four different favors of jam at an annual farmers market. The graph to the right shows the number of jars of each type of jam he sold at the market during the first two years. Which favor of jam had the greatest increase in number of jars sold from Year 1 to Year 2?
 - A) Blueberry
 - B) Grape
 - C) Peach
 - D) Strawberry



5) In the *x*-*y* plane, a line passes though the points (3,5) and the origin (0,0). Which of the following is the equation for the line?

A)
$$y = \frac{3}{5}x + 5$$

B) $y = \frac{5}{3}x$
C) $y = x + \frac{5}{3}$
D) $y = \frac{5}{3}x + 3$

- 6) $A = \pi R^2$ If the value of R is 2π , what is the value of A?
 - A) 2π B) 3π C) $4\pi^3$ D) $2\pi^2$
7) The table gives the population of the five largest countries in the European Union in the year 2014. Which of the following is closest to the mean population of these countries?

A)	80.8	million
B)	64.3	million
C)	63.7	million
D)	60.8	million

Country	Approximate population (millions)
France	65.9
Germany	80.8
Italy	60.8
Spain	46.5
United Kingdom	64.3

8) Which of the following equations is equivalent to $\frac{|-6|+6}{9}$?

- A) 0 B) 9 C) $\frac{1}{9}$ D) $\frac{4}{3}$
- 9) A runner completes a 100 meter race in 20 seconds. If they ran at a constant speed, how fast were they running in meters per second?
 - A) 5 *m/s*
 - B) 100 *m/s*
 - C) 4 *m/s*
 - D) 20 *m/s*
- 10) A box has a square base. One side of that base is 4 meters in length and the box has a height of 5 meters. What is the volume of the box?
 - A) $80 m^2$
 - B) $80 m^3$
 - C) 20 m
 - D) $20 m^3$

- 11) Mike is following a recipe that makes 3 pounds of bread for every 10 cups of sugar used. If Mike wants to make 9 pounds of bread, how many cups of sugar should he use?
 - A) 20 *cups*
 - B) 30 *cups*
 - C) 10 *cups*
 - D) $\frac{1}{3}$ cup
- 12) Which of the following is equivalent to the expression

$$4(x-3) + 6(x+2x-3)$$

A) 30x - 22B) 22x - 30C) 22x + 30D) 14x - 30

Answers

1)	С
2)	С
3)	A
4)	A
5)	В
6)	C
7)	С
8)	D
9)	A
10)	B
11)	В
12)	В

10.2 Practice Test 2

Answer the following questions.

- 1) It took Nick 90 minutes to read three books. Which is an equivalent rate?
 - A) 90 minutes per book
 - B) 1 book per minute
 - C) 30 minutes per book
 - D) 60 minutes per book
- 2) Which of the following is equivalent to 2^{-3}
 - A) $\frac{1}{8}$ B) 8 C) 6 D) $\frac{1}{6}$
- 3) Which of the following expressions is equivalent to $(x^3 \cdot x^{-2})^5$
 - A) x^{-5} B) xC) x^{5} D) x^{-1}

4) The elevation at the summit of Mount Whitney is 4,418 meters above sea level. Climbers begin at a trailhead that has an elevation of 2,550 meters above sea level. What is the change in elevation, to the nearest foot, between the trailhead and the summit? (1 *foot* = 0.3048 *meters*)

A) 569 feet
B) 5,604 feet
C) 6,129 feet
D) 14,495 feet

- 5) $x^2 + 3y x = 17$ and x = 2. Solve for y.
 - A) y = 5B) y = 15C) y = 17D) y = 45
- 6) 6x + 2y = 2 x + 1 = yThese equations intersect on a graph. Determine the point at which these lines intersect.
 - A) (0,1)
 B) (1,0)
 C) (1,-2)
 - D) (-2,1)

7) Sets *L*, *M*, and *N* are shown below.

 $L = \{0, 20, 40, 80, 100\}$ $M = \{5, 10, 15, 20, 25\}$ $N = \{10, 20, 30, 40, 50\}$

Which of the following sets represents $L \cup (M \cap N)$?

- A) {0, 5, 10, 15, 20, 25, 30, 40, 50, 80, 100}
- B) {0, 10, 20, 40, 80, 100}
- C) $\{20, 40\}$
- D) {20}
- 8) Robert wants to sell 100 tickets to his show. 20 tickets will be reserved for sale at 5 dollars for kids and the remaining tickets at 10 dollars for adults. Robert makes a total of 500 dollars and sells 40 adult tickets. Did Robert sell all the tickets reserved for kids?
 - A) Yes
 - B) No
- 9) Triangle PQR lies in the x-y plane, and the coordinates of vertex Q are (2, -3). Triangle PQR is rotated 180° clockwise about the origin and then reflected across the y-axis to produce triangle P'Q'R', where vertex Q' corresponds to vertex Q of triangle PQR. What are the coordinates of Q'?
 - A) (-3, -2)
 - B) (3, -2)
 - C) (-2,3)
 - D) (2,3)



- 10) Water runs from a pump at a rate of 1.5 gallons per minute. At this rate, how long would it take to fill a tub with a 150-gallon capacity?
 - A) 10 minutes
 B) 100 minutes
 C) 225 minutes
 D) 2,250 minutes
- 11) The amount of money M, in dollars, Addison earns can be represented by the equation M = 12.5h + 11, where h is the number of hours Addison works. Which of the following is the best interpretation of the number 11 in the equation?
 - A) The amount of money, in dollars, Addison earns each hour
 - B) The total amount of money, in dollars, Addison earns after working h hours
 - C) The total amount of money, in dollars, Addison earns after working for one hour
 - D) The amount of money, in dollars, Addison earns in addition to an hourly wage
- 12) Solve the following equation for x.

 $x^{2} - x - 6 = 0$ A) x = -3, 3B) x = -2, -3C) x = -2, 3D) x = -1, 6

Answers

1)	С
2)	A
3)	C
4)	С
5)	A
6)	Α
7)	В
8)	Α
9)	D
10)	B
11)	D
12)	С

10.3 Practice Test 3

Answer the following questions.

- 1) Which of the following is an equation of the line that passes through the point (0,0) and is perpendicular to the line shown to the right?
 - A) $y = \frac{5}{4}x$ B) $y = \frac{5}{4}x + 3$ C) $y = -\frac{4}{5}x$ D) $y = -\frac{4}{5}x + 3$



- 2) The linear equation is in the form ax + by = c, where a, b, and c are constants. If the line is graphed in the x-y plane and passes through the origin (0,0), which of the following constants must equal zero?
 - A) a
 B) b
 C) c
 D) It cannot be determined
- 3) Find x when $\frac{x}{10} = \frac{2}{5}$
 - A) x = 3
 B) x = 4
 C) x = 5
 D) x = 6

- 4) A farmer wants to fence off a rectangular area of 400 square feet on their farm and wants the side closest to their house to be 16 feet long. How many feet of fencing do they need to purchase to complete the project?
 - A) 25 feet
 B) 20 feet
 C) 82 feet
 D) 80 feet
- 5) Which equation is represented by the table of values shown to the right?
 - A) y = 3xB) y = 3x + 1C) y = 1x + 5D) y = 3x + 2

x	у
1	5
2	7
3	11
4	14
5	17

6) What is a value of x that satisfies the inequality?

 $18 < 2x^2 < 50$

- A) x = 4B) x = 5C) x = 20D) x = 25
- 7) Solve the following equation for x.

$$\frac{3}{2}x + \frac{2}{3} = \frac{1}{2}x$$

A) $x = \frac{2}{3}$ B) $x = \frac{1}{2}$ C) $x = -\frac{2}{3}$ D) $x = -\frac{1}{2}$

- 8) Multiply 0.25 by 0.2.
 - A) 0.05B) 0.5C) 5.0
 - D) 2.5
- 9) The histogram below shows the height (in cm) distribution of 30 people. How many people have heights between 150 and 180 cm?



Heights of 30 People

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- 10) For a triangle, the sides *a*, *b*, and *c* are related by the equation $a^2 + b^2 = c^2$. If a = 4 and $c = \sqrt{41}$, what is the length of side *b*?
 - A) b = 11B) b = 7C) b = 5D) b = 3



11) Solve the system of equations.

$$x + y = 7$$
 and $x + 2y = 11$

A) x = 0, y = 1B) x = 3, y = 2C) x = 3, y = 4D) x = 4, y = 4

12) Which of the following is a factor of both $x^2 - x - 6$ and $x^2 - 5x + 6$? A) x + 2B) x + 3C) x - 3

D) *x* – 2

Answers

1)	Α
2)	С
3)	В
4)	С
5)	D
6)	A
7)	C
8)	Α
9)	В
10)	С
11)	С
12)	С