METROPOLITAN STATE UNIVERSITY OF DENVER

NEXT-GENERATION MATH ACCUPLACER TEST REVIEW BOOKLET

Next Generation

Advanced Algebra and Functions

Property of MSU Denver Tutoring Center

Table of Contents

Each subsection in this practice book is followed by practice problems and answers. Make sure to check your answers!

This practice book contains a lot of material. The intention of including so much information is to make sure you're prepared for the test. You can start at the very beginning and work your way up, or you can focus on specific sections you need help with. Make sure you ask a tutor for help if you aren't sure about something!

<u>Sections with the Δ symbol at the top of the page are more likely to appear on the test!</u>

$$\Delta$$

About

Here is some information about the MSU Tutoring Center and the Placement Test.

MSU Denver Tutoring Center Mission

The mission of the MSU Denver Tutoring Center is to increase student persistence by offering free student-centered academic support across disciplines.
The Tutoring Center is not only for students who are having difficulty with course material, but also for students who are looking to excel. Whether your goal is to catch up, keep up, or do better in your studies, the Tutoring Center is here to help you by offering free one-on-one and group academic support in a variety of subjects. We promote an environment that is welcoming to the diverse student body of MSU Denver by providing professionally trained tutors who are competent in subject material and areas such as diversity, learning styles, and communication.

What is the Next-Generation Accuplacer?

The Next-Generation Advanced Algebra and Functions placement test is a computer adaptive assessment of test-takers' ability for selected mathematics content. Questions will focus on a range of topics, including a variety of equations and functions, including linear, quadratic, rational, radical, polynomial, and exponential. Questions will also delve into some geometry and trigonometry concepts. In addition, questions may assess a student's math ability via computational or fluency skills, conceptual understanding, or the capacity to apply mathematics presented in a context. All questions are multiple choice in format and appear discretely (stand-alone) across the assessment. The following knowledge and skill categories are assessed:

- Linear equations
- Linear applications
- Factoring
- Quadratics
- Functions
- Radical and rational equations
- Polynomial equations
- Exponential and logarithmic equations
- Geometry concepts
- Trigonometry

Test Taking Tips

- Take your time on the test. Students tend to get higher scores the longer they spend finding and double checking their answers.
- If a calculator is needed for a specific question, a calculator icon will appear in the top right corner. You may not bring your own calculator in to the test.
- Read carefully! Pay careful attention to how questions are worded and what they want you to find.
- Make use of the scratch paper provided- you are going to make mistakes if you try to solve problems in your head.
- The test is multiple choice, so use that to your advantage. If you can't remember how to approach a problem, maybe you'll find the correct answer by plugging the multiple choice answers into the original question!
- If you don't remember some of the rules for exponents, radicals, or other operations, try constructing a simpler example that you know better and solve that one. The rules may become clear to you once you try the simple example.
- Another strategy for remembering the many rules involved with operations is to break things down to their smaller pieces. Under pressure, it's easier to see what's going on with $x \cdot x \cdot x \cdot x$ instead of x^4.
- The test uses an algorithm called *branching*. This means that it changes its difficulty level each time you answer a question, depending on how well you've done. If you are getting questions wrong, the test will make itself easier. If you are getting questions correct, the test will make itself harder. It's a good sign if the test seems to be getting more challenging. (You earn more points for more difficult questions).
- Eat before you take the test. This will help you concentrate better.
- Get good sleep the night before you take the test. If you don't sleep well the night before, postpone it.
- Don't be intimidated by the questions or the test in general. It's okay to have anxiety- recognize it and do your best to relax so you can concentrate.
- Review for this test right before you go to sleep at night.

1

Integers and Rational Numbers

1.1 Useful Memorization

A calculator will not be allowed on the test, so make sure you have some of this memorized or know how to calculate it!

Multiplication Table

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Perfect Squares

$1^2 = 1$ $\quad$ $4^2 = 16$ $\quad$ $7^2 = 49$ $\quad$ $10^2 = 100$

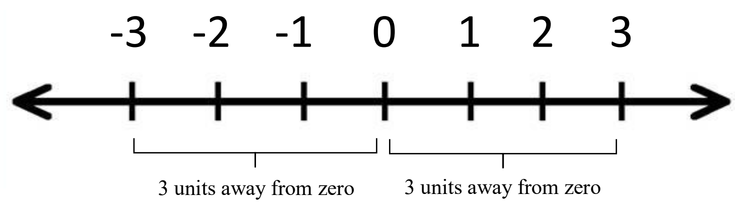
$2^2 = 4$ $\quad$ $5^2 = 25$ $\quad$ $8^2 = 64$ $\quad$ $11^2 = 121$

$3^2 = 9$ $\quad$ $6^2 = 36$ $\quad$ $9^2 = 81$ $\quad$ $12^2 = 144$

1.2 Absolute Value

The absolute value $| \ |$ functions as a grouping and an operation. It groups things together like parentheses, and it also changes negative numbers into positive numbers.

One way to think about the absolute value function is a measurement of distance. Consider $|3|$ and $|-3|$. How far away is 3 from zero on a number line? How far away is -3 from zero on a number line?



Since the number 3 is three units away from zero, $|3| = 3$. Since the number -3 is three units away from zero, $|-3| = 3$.

Example

An easy way to think about absolute value is that it turns negative numbers positive, and keeps positive numbers positive. Once the absolute value has been evaluated (turned positive), do not continue to write the vertical bars $| \ |$.

$$|-2| = 2 \qquad\qquad |2| = 2$$

Practice Problems

Evaluate the following.

1) $|-2|$
2) $|2|$
3) $|4|$
4) $|-5|$
5) $|-12|$
6) $|370|$
7) $|-160|$
8) $|0|$
9) $|5-3|$
10) $|12-1|$
11) $|3-8|$
12) $|9-16|$
13) $-|6|$
14) $-|-1|$
15) $|1 \div 3|$
16) $|-5| + |1|$
17) $|-3| + |-4|$
18) $|-9| - |6|$
19) $|-7+4|$
20) $-|-7+4|$
21) $-|-1-13|$

Answers

1)	2	-2 is two units away from zero.
2)	2	2 is two units away from zero.
3)	4	4 is four units away from zero.
4)	5	-5 is five units away from zero.
5)	12	-12 is twelve units away from zero.
6)	370	370 is 370 units away from zero.
7)	160	-160 is 160 units away from zero.
8)	0	0 is zero units away from itself.
9)	2	$\lvert 5-3\rvert = \lvert 2\rvert$, and 2 is two units away from zero.
10)	11	$\lvert 12-1\rvert = \lvert 11\rvert$, and 11 is eleven units away from zero.
11)	5	$\lvert 3-8\rvert = \lvert -5\rvert$, and -5 is five units away from zero.
12)	7	$\lvert 9-16\rvert = \lvert -7\rvert$, and -7 is seven units away from zero.
13)	-6	6 is six units away from zero and the negative is outside of the absolute value, so $-\lvert 6\rvert = -6$.
14)	-1	-1 is one unit away from zero and there is a negative outside of the absolute value, so $-\lvert -1\rvert = -1$.
15)	$\frac{1}{3}$	$\lvert 1 \div 3\rvert = \left\lvert \frac{1}{3}\right\rvert$, and $\frac{1}{3}$ is one third of a unit away from zero.
16)	6	$\lvert -5\rvert + \lvert 1\rvert = 5 + 1 = 6$
17)	7	$\lvert -3\rvert + \lvert -4\rvert = 3 + 4 = 7$
18)	3	$\lvert -9\rvert - \lvert 6\rvert = 9 - 6 = 3$
19)	3	$\lvert -7+4\rvert = \lvert -3\rvert$, and -3 is three units away from zero.
20)	-3	$-\lvert -7+4\rvert = -\lvert -3\rvert$, -3 is three units away from zero, and there is a negative outside of the absolute value, so $-\lvert -3\rvert = -3$.
21)	-14	$-\lvert -1-13\rvert = -\lvert -14\rvert$, -14 is fourteen units away from zero, and there is a negative outside of the absolute value, so $-\lvert -14\rvert = -14$.

1.3 Order of Operations

The different operations in mathematical expressions (addition, subtraction, multiplication, division, exponentiation, and grouping) must be evaluated in a specific order. This structure gives us consistent answers when we simplify expressions.

Why is it needed?

Consider the expression $4 \times 2 + 1$. If multiplication is evaluated before addition, $4 \times 2 + 1$ becomes $8 + 1$ which then becomes 9. If addition is evaluated before multiplication, $4 \times 2 + 1$ becomes 4×3 which then becomes 12. Since $9 \neq 12$, evaluating in any order we want doesn't give us consistent answers.

The Order of Operations

1) Parentheses (evaluate the inside) $\quad (\,)\ [\,]\ \{\,\}\ |\,|$
2) Exponents $\quad x^2 \leftarrow \textit{This number}$
3) Multiplication and Division (evaluate from left to right) $\quad \times\ \div$
4) Addition and Subtraction (evaluate from left to right) $\quad +\ -$

Notice that Multiplication and Division share the third spot and Addition and Subtraction share the fourth spot. In these cases, evaluate from left to right

PEMDAS is the mnemonic used to remember the Order of Operations. Start with Parentheses and end with addition and subtraction. (**P**arentheses, **E**xponents, **M**ultiplication, **D**ivision, **A**ddition, and **S**ubtraction).

Note about exponents

Pay close attention to notation with exponents! The position of a negative sign can change an entire problem.

$$-2^2 = -(2)(2) = -4$$

$$(-2)^2 = (-2)(-2) = 4$$

In the first expression, the negative is sitting outside of the exponent operation. In the second expression, it is included and cancels out.

Note about absolute value

The absolute value parentheses | | function in two ways. They group an expression just like any other parentheses, but they also turn whatever simplified number they contain into the positive version of that number.

$$|-3| = 3 \quad \text{and} \quad |3| = 3$$

Note about radicals

Radicals are technically exponents (see section 1.8). So radicals will be evaluated at the same time as exponents:

$$\sqrt{11+5}+2$$

$$\sqrt{16}+2$$

$$4+2$$

$$6$$

Practice Problems

Evaluate the following.

1) $(5+3)\cdot 2$
2) $5+(3\cdot 2)$
3) $2\cdot(8+5)-2$
4) $2\cdot 8+5-2$
5) $(2+3)^2-|2-4|$
6) $5+(3\cdot 6+2)$
7) -2^2+3^2
8) $(-2)^2+3^2$
9) $\frac{(-2)^2\cdot 2}{2^3}$
10) $((5-3)^2)^2$
11) $\frac{(5\cdot 2)(6-3)}{6\div 2}$
12) $\frac{(4+2)^1+2^3}{|2|}$
13) $0\cdot(3+6-8)^2-14$

Answers

1)	16	$(5+3)\cdot 2 = 8\cdot 2{=}16$
2)	11	$5+(3\cdot 2) = 5+6 = 11$
3)	24	$2\cdot(8+5)-2 = 2\cdot 13-2 = 26-2 = 24$
4)	19	$2\cdot 8+5-2 = 16+5-2 = 21-2 = 19$
5)	23	$(2+3)^2-\lvert 2-4\rvert = (5)^2-\lvert -2\rvert = 25-2 = 23$
6)	25	$5+(3\cdot 6+2) = 5+(18+2) = 5+(20) = 25$
7)	5	$-2^2+3^2 = -4+9 = 5$
8)	13	$(-2)^2+3^2 = 4+9 = 13$
9)	1	$\frac{(-2)^2\cdot 2}{2^3} = \frac{4\cdot 2}{8} = \frac{8}{8} = 1$
10)	16	$((5-3)^2)^2 = ((2)^2)^2 = (4)^2 = 16$
11)	10	$\frac{(5\cdot 2)(6-3)}{6\div 2} = \frac{(10)(3)}{3} = \frac{30}{3} = 10$
12)	7	$\frac{(4+2)^1+2^3}{\lvert 2\rvert} = \frac{(6)^1+8}{2} = \frac{6+8}{2} = \frac{14}{2} = 7$
13)	-14	$0\cdot(3+6-8)^2-14 = 0\cdot(9-8)^2-14 = 0\cdot(1)^2-14 = 0\cdot 1-14 = 0-14 = -14$

1.4 Substitution

Simply explained, substitution is when you replace the letters in an expression with numbers.

The Process

Replace the variables in the original expression with their assigned values.

Example

Evaluate x^2y where $x = 2$ and $y = 3$.

Starting point:	x^2y
Replace x with 2 and y with 3	$(2)^2 \cdot (3)$
Evaluate the exponent	$4 \cdot 3$
Multiply	12

Example

Evaluate x^2y where $x = -2$ and $y = -3$.

Starting point:	x^2y
Replace x with -2 and y with -3	$(-2)^2 \cdot (-3)$
Evaluate the exponent	$4 \cdot -3$
Multiply	-12

Note

It is important to plug in negative numbers carefully! A common mistake in the example above would be writing the second step as $-2^2 \cdot (-3)$ and therefore ending up with $-4 \cdot -3 = 12$ as your final answer instead of -12.

Practice Problems

Evaluate the following.

1) $|x|$ where $x = -3$
2) $|x^2|$ where $x = -3$
3) x^2y where $x = 3$ and $y = 2$
4) x^2y where $x = -4$ and $y = 2$
5) x^3 where $x = 2$
6) x^3 where $x = -2$
7) $-x^3$ where $x = 3$
8) $-x^3$ where $x = -3$
9) $(xyz)^2$ where $x = 3, y = -2$, and $z = \frac{1}{2}$
10) $y + \sqrt{x}$ where $x = 36$ and $y = -4$
11) $xy - \frac{3}{\sqrt{y}}$ where $x = \frac{1}{2}$ and $y = 4$
12) $\frac{x^2}{4} - \frac{y^2}{3}$ where $x = 3$ and $y = 2$
13) $\frac{5(x+h)-5(x)}{h}$ where $x = 3$ and $h = 4$
14) $\sqrt{\frac{s+r}{|s-r|}}$ where $r = 10$ and $s = 6$
15) $\frac{-b+\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}$ where $a = 4, b = 8$, and $c = 3$

Answers

1)	3	$\lvert x\rvert = \lvert -3\rvert = 3$
2)	9	$\lvert x^2\rvert = \lvert(-3)^2\rvert = \lvert 9\rvert = 9$
3)	18	$x^2y = (3)^2 2 = 9 \cdot 2 = 18$
4)	32	$x^2y = (-4)^2 2 = 16 \cdot 2 = 32$
5)	8	$x^3 = 2^3 = 2 \cdot 2 \cdot 2 = 8$
6)	-8	$x^3 = (-2)^3 = (-2)(-2)(-2) = -8$
7)	-27	$-x^3 = -(3)^3 = -(3)(3)(3) = -27$
8)	27	$-x^3 = -(-3)^3 = -(-3)(-3)(-3) = -(-27) = 27$
9)	9	$(xyz)^2 = (3 \cdot (-2) \cdot \frac{1}{2})^2 = (-3)^2 = 9$
10)	2	$y + \sqrt{x} = -4 + \sqrt{36} = -4 + 6 = 2$
11)	$\frac{1}{2}$	$xy - \frac{3}{\sqrt{y}} = \left(\frac{1}{2}\right) \cdot 4 - \frac{3}{\sqrt{4}} = \frac{4}{2} - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2}$
12)	$\frac{11}{12}$	$\frac{x^2}{4} - \frac{y^2}{3} = \frac{3^2}{4} - \frac{2^2}{3} = \frac{9}{4} - \frac{4}{3} = \left(\frac{3}{3}\right)\frac{9}{4} - \frac{4}{3}\left(\frac{4}{4}\right) = \frac{27}{12} - \frac{16}{12} = \frac{11}{12}$
13)	5	$\frac{5(x+h) - 5(x)}{h} = \frac{5(3+4) - 5(3)}{4} = \frac{5(7) - 5(3)}{4} = \frac{35 - 15}{4} = \frac{20}{4} = 5$
14)	2	$\sqrt{\frac{s+r}{\lvert s-r\rvert}} = \sqrt{\frac{6+10}{\lvert 6-10\rvert}} = \sqrt{\frac{16}{\lvert -4\rvert}} = \sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}} = \frac{4}{2} = 2$
15)	$-\frac{1}{2}$	$\frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-8 + \sqrt{(8)^2 - 4 \cdot 4 \cdot 3}}{2 \cdot 4} = \frac{-8 + \sqrt{64 - 48}}{8} = \frac{-8 + \sqrt{16}}{8} = \frac{-8 + 4}{8}$ $= \frac{-4}{8} = -\frac{1}{2}$

1.5 Rules of Fractions

Fractions can be reduced, added, subtracted, multiplied, and divided.

Multiplication

Rule

$$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$$

Example

$$\frac{2}{3} * \frac{1}{2} = \frac{2 * 1}{3 * 2} = \frac{2}{6} = \frac{1}{3}$$

Division

Rule

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c} = \frac{a * d}{b * c}$$

Example

$$\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} * \frac{2}{1} = \frac{2 * 2}{3 * 1} = \frac{4}{3}$$

Addition and Subtraction

Rule

$$\frac{a}{b} + \frac{c}{d} = \left(\frac{d}{d}\right)\frac{a}{b} + \frac{c}{d}\left(\frac{b}{b}\right) = \frac{a * d + b * c}{b * d}$$

Example

$$\frac{2}{3} + \frac{1}{2} = \left(\frac{2}{2}\right)\frac{2}{3} + \frac{1}{2}\left(\frac{3}{3}\right) = \frac{2 * 2 + 1 * 3}{2 * 3}$$

$$= \frac{4 + 3}{6} = \frac{7}{6}$$

The rule above works for addition or subtraction. Just replace the + with a – to get:

$$\frac{a}{b} - \frac{c}{d} = \left(\frac{d}{d}\right)\frac{a}{b} - \frac{c}{d}\left(\frac{b}{b}\right) = \frac{a * d - b * c}{b * d}$$

Practice Problems

1) $\frac{1}{2} \cdot \frac{3}{7}$

2) $\frac{9}{10} \cdot \frac{2}{10}$

3) $\frac{-3}{5} \cdot \frac{-15}{9}$

4) $\frac{1}{3} \div \frac{1}{9}$

5) $\frac{3}{4} \div \frac{1}{2}$

6) $8\left(\frac{1}{5}\right)$

7) $\frac{3}{7} + \frac{2}{3}$

8) $\frac{3}{7} - \frac{2}{3}$

9) $\frac{5}{4} + \frac{5}{3}$

10) $\frac{4}{9} - \frac{3}{2}$

11) $\frac{5}{4} + \frac{7}{8}\left(\frac{8}{7}\right)$

Answers

1)	$\frac{3}{14}$	$\frac{1}{2}\cdot\frac{3}{7}=\frac{1\cdot 3}{2\cdot 7}=\frac{3}{14}$
2)	$\frac{9}{50}$	$\frac{9}{10}\cdot\frac{2}{10}=\frac{9\cdot 2}{10\cdot 10}=\frac{18}{100}=\frac{9}{50}$
3)	1	$\frac{-3}{5}\cdot\frac{-15}{9}=\frac{-3\cdot -15}{5\cdot 9}=\frac{45}{45}=1$
4)	3	$\frac{1}{3}\div\frac{1}{9}=\frac{1}{3}\cdot\frac{9}{1}=\frac{1\cdot 9}{3\cdot 1}=\frac{9}{3}=3$
5)	$\frac{3}{2}$	$\frac{3}{4}\div\frac{1}{2}=\frac{3}{4}\cdot\frac{2}{1}=\frac{3\cdot 2}{4\cdot 1}=\frac{6}{4}=\frac{3}{2}$
6)	$\frac{8}{5}$	$8\left(\frac{1}{5}\right)=\frac{8}{1}\cdot\frac{1}{5}=\frac{8\cdot 1}{1\cdot 5}=\frac{8}{5}$
7)	$\frac{23}{21}$	$\frac{3}{7}+\frac{2}{3}=\left(\frac{3}{3}\right)\frac{3}{7}+\frac{2}{3}\left(\frac{7}{7}\right)=\frac{3\cdot 3}{3\cdot 7}+\frac{2\cdot 7}{3\cdot 7}=\frac{9}{21}+\frac{14}{21}=\frac{9+14}{21}=\frac{23}{21}$
8)	$-\frac{5}{21}$	$\frac{3}{7}-\frac{2}{3}=\left(\frac{3}{3}\right)\frac{3}{7}-\frac{2}{3}\left(\frac{7}{7}\right)=\frac{3\cdot 3}{3\cdot 7}-\frac{2\cdot 7}{3\cdot 7}=\frac{9}{21}-\frac{14}{21}=\frac{9-14}{21}=-\frac{5}{21}$
9)	$\frac{35}{12}$	$\frac{5}{4}+\frac{5}{3}=\left(\frac{3}{3}\right)\frac{5}{4}+\frac{5}{3}\left(\frac{4}{4}\right)=\frac{3\cdot 5}{3\cdot 4}+\frac{5\cdot 4}{3\cdot 4}=\frac{15}{12}+\frac{20}{12}=\frac{15+20}{21}=\frac{35}{12}$
10)	$-\frac{19}{21}$	$\frac{4}{9}-\frac{3}{2}=\left(\frac{2}{2}\right)\frac{4}{9}-\frac{3}{2}\left(\frac{9}{9}\right)=\frac{2\cdot 4}{2\cdot 9}-\frac{3\cdot 9}{2\cdot 9}=\frac{8}{18}-\frac{27}{18}=\frac{8-27}{21}=-\frac{19}{21}$
11)	$\frac{9}{4}$	$\frac{5}{4}+\frac{7}{8}\left(\frac{8}{7}\right)=\frac{5}{4}+\frac{7\cdot 8}{8\cdot 7}=\frac{5}{4}+\frac{56}{56}=\left(\frac{14}{14}\right)\frac{5}{4}+\frac{56}{56}=\frac{70}{56}+\frac{56}{56}=\frac{70+56}{56}=\frac{126}{56}=\frac{9}{4}$

1.6 Mixed Fractions

The following are four definitions in regards to fractions.

Terminology

A *whole number* is a fraction where the denominator is 1.

$$\frac{any\ number}{1} \rightarrow \frac{4}{1} = 4$$

Terminology

A *proper fraction* is a fraction where the numerator is smaller than the denominator.

$$\frac{numerator\ smaller}{denominator\ larger} \rightarrow \frac{2}{3}$$

Terminology

An *improper fraction* is a fraction where the numerator is larger than the denominator.

$$\frac{numerator\ larger}{denominator\ smaller} \rightarrow \frac{7}{2}$$

Terminology

A *mixed fraction* is a whole number and a proper fraction combined.

$$whole\ number\frac{numerator\ smaller}{denominator\ larger}$$

$$\rightarrow 7\frac{2}{3}$$

Write Improper Fraction as Mixed Fraction

Starting point:	$\frac{7}{2}$
Re-write as the sum of fractions:	$= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2}$
Simplify to whole numbers:	$= 1 + 1 + 1 + \frac{1}{2}$
Combine:	$= 3 + \frac{1}{2}$
Write the whole number next to the proper fraction:	$= 3\frac{1}{2}$

Write Mixed Fraction as Improper Fraction

Starting point:	$2\frac{3}{7}$
Re-write as the sum of fraction and whole number:	$= 2 + \frac{3}{7}$
Expand:	$= 1 + 1 + \frac{3}{7}$
Write whole numbers as fractions:	$= \frac{7}{7} + \frac{7}{7} + \frac{3}{7}$
Add across the top $(7 + 7 + 3)$ and keep the bottom the same:	$= \frac{17}{7}$

Practice Problems

Write the following improper fractions as mixed fractions.

1) $\frac{8}{7}$
2) $\frac{9}{2}$
3) $\frac{3}{2}$
4) $\frac{11}{3}$
5) $\frac{23}{3}$
6) $\frac{17}{5}$
7) $\frac{17}{6}$

Write the following mixed fractions as improper fractions.

8) $1\frac{1}{2}$
9) $4\frac{1}{3}$
10) $3\frac{3}{10}$
11) $6\frac{1}{2}$
12) $2\frac{5}{9}$
13) $7\frac{2}{3}$
14) $1\frac{4}{7}$

Answers

1)	$1\frac{1}{7}$	$\frac{8}{7}=\frac{7}{7}+\frac{1}{7}=1+\frac{1}{7}=1\frac{1}{7}$
2)	$4\frac{1}{2}$	$\frac{9}{2}=\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{1}{2}=1+1+1+1+\frac{1}{2}=4+\frac{1}{2}=4\frac{1}{2}$
3)	$1\frac{1}{2}$	$\frac{3}{2}=\frac{2}{2}+\frac{1}{2}=1+\frac{1}{2}=1\frac{1}{2}$
4)	$3\frac{2}{3}$	$\frac{11}{3}=\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{2}{3}=1+1+1+\frac{2}{3}=3+\frac{2}{3}=3\frac{2}{3}$
5)	$7\frac{2}{3}$	$\frac{23}{3}=\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{2}{3}=1+1+1+1+1+1+1+\frac{2}{3}=7+\frac{2}{3}=7\frac{2}{3}$
6)	$3\frac{2}{5}$	$\frac{17}{5}=\frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{2}{5}=1+1+1+\frac{2}{5}=3+\frac{2}{5}=3\frac{2}{5}$
7)	$2\frac{5}{6}$	$\frac{17}{6}=\frac{6}{6}+\frac{6}{6}+\frac{5}{6}=1+1+\frac{5}{6}=2+\frac{5}{6}=2\frac{5}{6}$
8)	$\frac{3}{2}$	$1\frac{1}{2}=1+\frac{1}{2}=\frac{2}{2}+\frac{1}{2}=\frac{3}{2}$
9)	$\frac{13}{3}$	$4\frac{1}{3}=4+\frac{1}{3}=\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{1}{3}=\frac{13}{3}$
10)	$\frac{33}{10}$	$3\frac{3}{10}=3+\frac{3}{10}=\frac{10}{10}+\frac{10}{10}+\frac{10}{10}+\frac{3}{10}=\frac{33}{10}$
11)	$\frac{13}{2}$	$6\frac{1}{2}=6+\frac{1}{2}=\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{1}{2}=\frac{13}{2}$
12)	$\frac{23}{9}$	$2\frac{5}{9}=2+\frac{5}{9}=\frac{9}{9}+\frac{9}{9}+\frac{5}{9}=\frac{23}{9}$
13)	$\frac{23}{3}$	$7\frac{2}{3}=7+\frac{2}{3}=\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{2}{3}=\frac{23}{3}$
14)	$\frac{11}{7}$	$1\frac{4}{7}=1+\frac{4}{7}=\frac{7}{7}+\frac{4}{7}=\frac{11}{7}$

1.7 Rules of Exponents

An exponent is a base raised to a power. A base 'a' raised to the power of 'n' is equal to the multiplication of a, n times:

$$a^n = a \times a \times \ldots \times a \qquad [n \text{ times}]$$

a is the base and n is the exponent.

Examples

$\boldsymbol{x^1 = x}$	$2^1 = 2$
$\boldsymbol{x^2 = x \cdot x}$	$2^2 = 2 \cdot 2$
$\boldsymbol{x^3 = x \cdot x \cdot x}$	$2^3 = 2 \cdot 2 \cdot 2$
$\boldsymbol{x^4 = x \cdot x \cdot x \cdot x}$	$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$
$\boldsymbol{x^5 = x \cdot x \cdot x \cdot x \cdot x}$	$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Rules

$a^m \cdot a^n = a^{m+n}$
$a^m \cdot b^m = (a * b)^m$
$\frac{a^m}{a^n} = a^{m-n}$
$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$(a * b)^m = a^m * b^m$
$(a^m)^n = a^{m*n}$
$a^{-n} = \frac{1}{a^n}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$a^0 = 1$
$0^n = 0, \qquad for\ n > 0$

Practice Problems

Simplify the following expressions.

1) $2^2 + 3^1$

2) $3^2 \cdot 3^2$

3) $3^2 \cdot 3^{-2}$

4) $\frac{(3^2)^3 - (2^2)^2}{31}$

5) $-2^4 + (-2)^4 - 2^1$

6) $\left(\frac{3}{2}\right)^3$

7) $(3^2 \cdot 2^3) \cdot 3^{-3} + (3 \div 3^3)$

8) $\frac{2^5}{-2^2} + \frac{2}{2^3}$

9) $(2^3)^{-2}(2^2 \cdot 1^3)^2$

10) $\frac{x^5}{x^2}$

11) $\frac{x^3y^2}{x^2y^5}$

12) $(2x^2y^3)^0$

13) $\frac{(3x^4y^{-2})^{-3}}{(2x^3y^2)^{-2}}$

Answers

1)	7	$2^2 + 3^1 = 4 + 3 = 7$
2)	81	$3^2 \cdot 3^2 = 9 \cdot 9 = 81$
3)	1	$3^2 \cdot 3^{-2} = 3^2 \cdot \frac{1}{3^2} = \frac{3^2}{3^2} = 1 \quad or \quad 3^2 \cdot 3^{-2} = 3^{2-2} = 3^0 = 1$
4)	23	$\frac{(3^2)^3 - (2^2)^2}{31} = \frac{3^6 - 2^4}{31} = \frac{729-16}{31} = \frac{713}{31} = 23$
5)	-2	$-2^4 + (-2)^4 - 2^1 = -16 + 16 - 2 = 0 - 2 = -2$
6)	$\frac{27}{8}$	$\left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$
7)	$\frac{25}{9}$	$(3^2 \cdot 2^3) \cdot 3^{-3} + (3 \div 3^3) = (9 \cdot 8) \cdot \frac{1}{3^3} + \left(\frac{3}{3^3}\right) = \frac{72}{27} + \frac{3}{27} = \frac{75}{27} = \frac{25}{9}$
8)	$\frac{33}{4}$	$\frac{2^5}{(-2)^2} + \frac{2}{2^3} = \frac{32}{4} + \frac{2}{8} = \left(\frac{2}{2}\right)\frac{32}{4} + \frac{2}{8} = \frac{64}{8} + \frac{2}{8} = \frac{64+2}{8} = \frac{66}{8} = \frac{33}{4}$
9)	$\frac{1}{4}$	$(2^3)^{-2}(2^2 \cdot 1^3)^2 = 2^{-6}(4 \cdot 1)^2 = \frac{1}{2^6} \cdot (4)^2 = \frac{16}{64} = \frac{1}{4}$
10)	x^3	$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{x \cdot x \cdot x}{1} = x^3 \quad or \quad \frac{x^5}{x^2} = x^{5-2} = x^3$
11)	$\frac{x}{y^3}$	$\frac{x^3y^2}{x^2y^5} = \frac{x \cdot x \cdot x \cdot y \cdot y}{x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y} = \frac{x}{y \cdot y \cdot y} = \frac{x}{y^3} \quad or \quad \frac{x^3y^2}{x^2y^5} = x^{3-2}y^{2-5} = x^1y^{-3} = x\left(\frac{1}{y^3}\right) = \frac{x}{y^3}$
12)	1	$(2x^2y^3)^0 = 2^0x^0y^0 = 1 \cdot 1 \cdot 1 = 1 \quad or \quad (2x^2y^3)^0 = 1$ (anything to the power of zero is one)
13)	$\frac{4y^{10}}{27x^6}$	$\frac{(3x^4y^{-2})^{-3}}{(2x^3y^2)^{-2}} = \frac{3^{-3}x^{-12}y^6}{2^{-2}x^{-6}y^{-4}} = \frac{2^2x^6y^4y^6}{3^3x^{12}} = \frac{2^2x^6y^{10}}{3^3x^{12}} = \frac{4x^6y^{10}}{27x^{12}} = \frac{4y^{10}}{27x^6}$

1.8 Simplifying Radicals

A radical is anything with the root symbol $\sqrt{\ }$. Radicals are **fractional exponents**. The notation is very different, but they follow the same basic rules as regular exponents! With practice, you will be able to see the similarities. The following shows how radicals work on a basic level and how to simplify them.

Notation

$\sqrt{x} = \sqrt[2]{x} = x^{1/2}$	$4^{1/2} = \sqrt[2]{4} = \sqrt[2]{2 \cdot 2} = 2$
$\sqrt[3]{x} = x^{1/3}$	$8^{1/3} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2$
$\sqrt[4]{x} = x^{1/4}$	$16^{1/4} = \sqrt[4]{16} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = 2$
$\sqrt[5]{x} = x^{1/5}$	$32^{1/5} = \sqrt[5]{32} = \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2$
$\sqrt[6]{x} = x^{1/6}$	$64^{1/6} = \sqrt[6]{64} = \sqrt[6]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2$

Simplification Example

A radical is fully simplified when the radicand (the part under the radical) has no square factors. For instance, $\sqrt{40} = \sqrt{4 \cdot 5 \cdot 2}$, and since 4 is a square number, $\sqrt{40}$ is not simplified. From here, use the rules of radicals to split the root apart and simplify:

$$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$$

So $2\sqrt{10}$ is the most simplified form of $\sqrt{40}$.

Pattern-Based Approach

Another way to simplify is to factor the radicand completely. For example, $\sqrt{40} = \sqrt{2 \cdot 2 \cdot 2 \cdot 5}$. Notice there are three twos. The type of radical we are using is a square root ($\sqrt{2 \cdot 2 \cdot 2 \cdot 5} = \sqrt[2]{2 \cdot 2 \cdot 2 \cdot 5}$). Since that *two* is there, we find pairs of *two* and represent the two outside, leaving behind the unpaired numbers under the radical. $\sqrt[2]{\mathbf{2 \cdot 2} \cdot 2 \cdot 5} = \mathbf{2} \cdot \sqrt[2]{2 \cdot 5}$. There are more examples on the next page.

Pattern-Based Approach Examples

$\sqrt{40} = \sqrt{\mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 5} = \mathbf{2} \cdot \sqrt{2 \cdot 5} = 2\sqrt{10}$	groups of 2 because of $\sqrt{}$
$\sqrt{72} = \sqrt{\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot \mathbf{3} \cdot 2} = \mathbf{2} \cdot \mathbf{3} \cdot \sqrt{2} = 6\sqrt{2}$	groups of 2 because of $\sqrt{}$
$\sqrt{16} = \sqrt{\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2}} = \mathbf{2} \cdot \mathbf{2} = 4$	groups of 2 because of $\sqrt{}$
$\sqrt[3]{40} = \sqrt[3]{\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 5} = \mathbf{2} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$	groups of 3 because of $\sqrt[3]{}$
$\sqrt[3]{8} = \sqrt[3]{\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2}} = \mathbf{2} = 2$	groups of 3 because of $\sqrt[3]{}$
$\sqrt[3]{250} = \sqrt[3]{\mathbf{5} \cdot \mathbf{5} \cdot \mathbf{5} \cdot 2} = \mathbf{5} \cdot \sqrt[3]{2} = 5\sqrt[3]{2}$	groups of 3 because of $\sqrt[3]{}$
$\sqrt[4]{16} = \sqrt[4]{\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2}} = \mathbf{2} = 2$	groups of 4 because of $\sqrt[4]{}$
$\sqrt[4]{32} = \sqrt[4]{\mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 2} = \mathbf{2} \cdot \sqrt[4]{2} = 2\sqrt[4]{2}$	groups of 4 because of $\sqrt[4]{}$

If you factor the radicand completely and can't find any pairs, then the radical is already as simple as it can be. For example, $\sqrt{30} = \sqrt{2 \cdot 3 \cdot 5}$. Since there are no pairs, $\sqrt{30}$ is the most simplified form.

Practice Problems

Simplify the following radicals.

1) $\sqrt{20}$
2) $\sqrt{16}$
3) $\sqrt{24}$
4) $\sqrt{18}$
5) $\sqrt{180}$
6) $\sqrt{12}$
7) $\sqrt{28}$
8) $\sqrt{50}$
9) $\sqrt{45}$
10) $\sqrt{98}$
11) $\sqrt{48}$
12) $\sqrt{300}$
13) $\sqrt{150}$
14) $\sqrt{80}$
15) $\frac{\sqrt{20}}{2}$

Answers

1)	$2\sqrt{5}$	$\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$
2)	4	$\sqrt{16} = \sqrt{4 \cdot 4} = 4$
3)	$2\sqrt{6}$	$\sqrt{24} = \sqrt{3 \cdot 2 \cdot 2 \cdot 2} = 2\sqrt{6}$
4)	$3\sqrt{2}$	$\sqrt{18} = \sqrt{3 \cdot 3 \cdot 2} = 3\sqrt{2}$
5)	$6\sqrt{5}$	$\sqrt{180} = \sqrt{3 \cdot 3 \cdot 2 \cdot 2 \cdot 5} = 6\sqrt{5}$
6)	$2\sqrt{3}$	$\sqrt{12} = \sqrt{3 \cdot 2 \cdot 2} = 2\sqrt{3}$
7)	$2\sqrt{7}$	$\sqrt{28} = \sqrt{2 \cdot 2 \cdot 7} = 2\sqrt{7}$
8)	$5\sqrt{2}$	$\sqrt{50} = \sqrt{2 \cdot 5 \cdot 5} = 5\sqrt{2}$
9)	$3\sqrt{5}$	$\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5} = 3\sqrt{5}$
10)	$7\sqrt{2}$	$\sqrt{98} = \sqrt{2 \cdot 7 \cdot 7} = 7\sqrt{2}$
11)	$4\sqrt{3}$	$\sqrt{48} = \sqrt{4 \cdot 4 \cdot 3} = 4\sqrt{3}$
12)	$10\sqrt{3}$	$\sqrt{300} = \sqrt{10 \cdot 10 \cdot 3} = 10\sqrt{3}$
13)	$5\sqrt{6}$	$\sqrt{150} = \sqrt{5 \cdot 5 \cdot 3 \cdot 2} = 5\sqrt{6}$
14)	$4\sqrt{5}$	$\sqrt{80} = \sqrt{4 \cdot 4 \cdot 5} = 4\sqrt{5}$
15)	$\sqrt{5}$	$\frac{\sqrt{20}}{2} = \frac{\sqrt{2 \cdot 2 \cdot 5}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

1.9 Rules of Radicals

Since radicals are fractional exponents, we can use rules of exponents to manipulate them. The following are some of the properties and notation with radicals.

Rules

$x^{\frac{a}{b}} = \sqrt[b]{x^a}$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$
$\sqrt[a]{x^a} = x$	$\sqrt[2]{x^2} = x^{\frac{2}{2}} = x^1 = x$
$\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{x \cdot y}$	$\sqrt[3]{x} \cdot \sqrt[3]{y} = \sqrt[3]{xy}$
$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$	$\sqrt[5]{\frac{x}{y}} = \frac{\sqrt[5]{x}}{\sqrt[5]{y}}$

Note On Multiplication And Division

If the radicals are not the same type (number), the multiplication and division rules do not apply.

Rule	Examples
$\sqrt[a]{x} \cdot \sqrt[b]{y}$ ***cannot combine***	$\sqrt[2]{x} \cdot \sqrt[3]{y}$ is already as simple as possible
$\frac{\sqrt[a]{x}}{\sqrt[b]{y}}$ ***cannot combine***	$\frac{\sqrt[5]{x}}{\sqrt[4]{y}}$ is already as simple as possible

Practice Problems

Evaluate the following radicals.

1) $\left(\sqrt{3}\right)^2$
2) $\sqrt{(3)^2}$
3) $\sqrt{2} \cdot \sqrt{15}$
4) $\sqrt{6} \cdot \sqrt{8}$
5) $\sqrt{\frac{4}{9}}$
6) $-\sqrt{\frac{6}{36}}$
7) $16^{\frac{2}{8}}$
8) $\left(\sqrt[3]{8}\right)^2$
9) $\frac{8}{3\sqrt{64}}$
10) $2\sqrt{7} + 5\sqrt{7}$
11) $\sqrt[3]{-125}$

Answers

1)	3	$\left(\sqrt{3}\right)^2 = 3^{\frac{2}{2}} = 3^1 = 3$
2)	3	$\sqrt{(3)^2} = 3^{\frac{2}{2}} = 3^1 = 3$
3)	$\sqrt{30}$	$\sqrt{2} \cdot \sqrt{15} = \sqrt{30}$
4)	$4\sqrt{3}$	$\sqrt{6} \cdot \sqrt{8} = \sqrt{6 \cdot 8} = \sqrt{48} = 4\sqrt{3}$
5)	$\frac{2}{3}$	$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$
6)	$-\frac{\sqrt{6}}{6}$	$-\sqrt{\frac{6}{36}} = -\frac{\sqrt{6}}{\sqrt{36}} = -\frac{\sqrt{6}}{6}$
7)	2	$16^{\frac{2}{8}} = \sqrt[8]{(16)^2} = \sqrt[8]{16 \cdot 16} = \sqrt[8]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2$
8)	4	$\left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$
9)	$\frac{1}{3}$	$\frac{8}{3\sqrt{64}} = \frac{8}{3 \cdot 8} = \frac{1}{3}$
10)	$7\sqrt{7}$	$2\sqrt{7} + 5\sqrt{7} = 7\sqrt{7}$
11)	-5	$\sqrt[3]{-125} = -5$

1.10 Rationalizing

When radicals appear in a fraction, we may have to use algebra to rearrange the fraction so that it is in the proper form. Radicals are allowed to be in the numerator, but not the denominator.

If a radical is in the denominator, the process of rearranging the fraction so that the radical is only in the numerator is called **rationalization**. First some terminology is given, then examples of two situations involving rationalization are outlined.

Terminology

A **binomial** is formed when two terms are added or subtracted from one another. $(x+y)$ is a binomial. $(2x-7)$ is also a binomial.

The **conjugate** of a binomial is created when the plus or minus sign in the *middle* of the binomial is changed to the opposite sign. $(x-y)$ is the conjugate of $(x+y)$. $(2x+7)$ is the conjugate of $(2x-7)$.

Example

Consider the following fraction. In order to rationalize a fraction with a radical in the denominator, multiply both the numerator and denominator by that same radical.

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{3\sqrt{5}}{\left(\sqrt{5}\right)^2} = \frac{3\sqrt{5}}{5}$$

Not rationalized

Multiply by fraction made up of the radical

The exponent and radical in the denominator cancel out

Rationalized

In other examples, reduce further if necessary.

Example

When a radical in the denominator is part of a binomial, multiply the numerator and denominator by the conjugate.

$$\frac{2}{7+\sqrt{5}} = \frac{2}{7+\sqrt{5}}\left(\frac{7-\sqrt{5}}{7-\sqrt{5}}\right) = \frac{2\left(7-\sqrt{5}\right)}{7^2-\left(\sqrt{5}\right)^2} = \frac{14-2\sqrt{5}}{49-5} = \frac{7-\sqrt{5}}{22}$$

Not rationalized

Multiply by fraction made up of the conjugate

The exponent and radical in the denominator cancel out

Reduce…

Rationalized

Practice Problems

Rationalize the following fractions.

1) $\frac{1}{\sqrt{2}}$
2) $\frac{1}{\sqrt{3}}$
3) $\frac{2}{\sqrt{5}}$
4) $\frac{4}{\sqrt{3}}$
5) $\frac{x}{\sqrt{2}}$
6) $\frac{1}{3+\sqrt{2}}$
7) $\frac{1}{1+\sqrt{3}}$
8) $\frac{5}{2-\sqrt{2}}$
9) $\frac{3}{4-\sqrt{3}}$
10) $\frac{2}{x+\sqrt{5}}$

Answers

1)	$\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{1\cdot\sqrt{2}}{\left(\sqrt{2}\right)^2}=\frac{\sqrt{2}}{2}$
2)	$\frac{\sqrt{3}}{3}$	$\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{1\cdot\sqrt{3}}{\left(\sqrt{3}\right)^2}=\frac{\sqrt{3}}{3}$
3)	$\frac{2\sqrt{5}}{5}$	$\frac{2}{\sqrt{5}}=\frac{2}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)=\frac{2\cdot\sqrt{5}}{\left(\sqrt{5}\right)^2}=\frac{2\sqrt{5}}{5}$
4)	$\frac{4\sqrt{3}}{3}$	$\frac{4}{\sqrt{3}}=\frac{4}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{4\cdot\sqrt{3}}{\left(\sqrt{3}\right)^2}=\frac{4\sqrt{3}}{3}$
5)	$\frac{x\sqrt{2}}{2}$	$\frac{x}{\sqrt{2}}=\frac{x}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{x\cdot\sqrt{2}}{\left(\sqrt{2}\right)^2}=\frac{x\sqrt{2}}{2}$
6)	$\frac{3-\sqrt{2}}{7}$	$\frac{1}{3+\sqrt{2}}=\frac{1}{3+\sqrt{2}}\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right)=\frac{3-\sqrt{2}}{3^2-\left(\sqrt{2}\right)^2}=\frac{3-\sqrt{2}}{9-2}=\frac{3-\sqrt{2}}{7}$
7)	$-\frac{1-\sqrt{3}}{2}$	$\frac{1}{1+\sqrt{3}}=\frac{1}{1+\sqrt{3}}\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)=\frac{1-\sqrt{3}}{1^2-\left(\sqrt{3}\right)^2}=\frac{1-\sqrt{3}}{1-3}=\frac{1-\sqrt{3}}{-2}=-\frac{1-\sqrt{3}}{2}$
8)	$\frac{10+5\sqrt{2}}{2}$	$\frac{5}{2-\sqrt{2}}=\frac{5}{2-\sqrt{2}}\left(\frac{2+\sqrt{2}}{2+\sqrt{2}}\right)=\frac{5\left(2+\sqrt{2}\right)}{2^2-\left(\sqrt{2}\right)^2}=\frac{10+5\sqrt{2}}{4-2}=\frac{10+5\sqrt{2}}{2}$
9)	$\frac{12+3\sqrt{3}}{13}$	$\frac{3}{4-\sqrt{3}}=\frac{3}{4-\sqrt{3}}\left(\frac{4+\sqrt{3}}{4+\sqrt{3}}\right)=\frac{3\left(4+\sqrt{3}\right)}{4^2-\left(\sqrt{3}\right)^2}=\frac{12+3\sqrt{3}}{16-3}=\frac{12+3\sqrt{3}}{13}$
10)	$\frac{2x-2\sqrt{5}}{x^2-5}$	$\frac{2}{x+\sqrt{5}}=\frac{2}{x+\sqrt{5}}\left(\frac{x-\sqrt{5}}{x-\sqrt{5}}\right)=\frac{2\left(x-\sqrt{5}\right)}{x^2-\left(\sqrt{5}\right)^2}=\frac{2x-2\sqrt{5}}{x^2-5}$

2

Linear Expressions and Equations

2.1 Simplifying Expressions

Some algebraic expressions can be reduced to a simpler form. Most of the time, the answers to a problem will be in the simplest form.

Use the basic rules of algebra to combine alike terms and simplify.

Example

Combine like terms when possible.

Starting Point:	$2y^3 \cdot 4y^2$
Group alike terms:	$2 \cdot 4 \cdot y^3 \cdot y^2$
Combine terms that can be combined:	$8 \cdot y^{3+2}$
Simplify exponent:	$8y^5$

Example

Combine like terms.

Starting Point:	$x^2 + 3x^2 + 2x^2 + x^3$
Combine based on number in front:	$6x^2 + x^3$

Example

Cancel terms that are in both the numerator and denominator of a fraction!

Starting Point:	$\dfrac{3x^2y^4}{6x^4y^3}$
Expand the expression:	$\dfrac{3\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y}{3\cdot 2\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y}$
Cancel out terms that appear on the top and bottom:	$\dfrac{\cancel{3}\cdot\cancel{x}\cdot\cancel{x}\cdot y\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}}{2\cdot\cancel{3}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}}$
Write without the cancelled terms:	$\dfrac{y}{2\cdot x\cdot x}$
Final answer:	$\dfrac{y}{2x^2}$

Example

Use the rules of exponents, radicals and fractions to simplify.

Starting Point:	$2x^{-3}$
Use rules of exponents:	$2\left(\dfrac{1}{x^3}\right)$
Multiply:	$\dfrac{2}{x^3}$

Example

Factor and use cancellation when you come across polynomials! Refer to Section 5.2 to learn more about factoring.

Starting Point:	$\frac{x^2+4x+4}{x^2-4}$
Factor the numerator and denominator:	$\frac{(x+2)(x+2)}{(x+2)(x-2)}$
Cancel out terms that appear on the top and bottom:	$\frac{\cancel{(x+2)}(x+2)}{\cancel{(x+2)}(x-2)}$
Write without the cancelled terms:	$\frac{(x+2)}{(x-2)}$
Final answer:	$\frac{x+2}{x-2}$

Practice Problems

Simplify the following algebraic expressions.

1) $x + 3x + 4x - 2x$
2) $-7 + 3x + 7 - 5x + 1$
3) $3x^2 \cdot 3x^4$
4) $2x \cdot x^{-2}$
5) $x \div x^3$
6) $x^2 \cdot x^1 + x^3$
7) $4x^3 - 8x^3 + x^3$
8) $\frac{x}{2} + \frac{x}{8}$
9) $\frac{x^2y^7}{2x^3y^4}$

Answers

1)	$6x$	$x + 3x + 4x - 2x = 4x + 4x - 2x = 8x - 2x = 6x$
2)	$1 - 2x$	$-7 + 3x + 7 - 5x + 1 = -7 + 7 + 1 - 5x + 3x = 1 - 5x + 3x = 1 - 2x$
3)	$9x^6$	$3x^2 \cdot 3x^4 = 3 \cdot 3 \cdot x^2 \cdot x^4 = 9 \cdot x^2 \cdot x^4 = 9 \cdot x^6 = 9x^6$
4)	$\frac{2}{x}$	$2x \cdot x^{-2} = 2x \cdot \frac{1}{x^2} = \frac{2x}{x^2} = \frac{2}{x}$
5)	$\frac{1}{x^2}$	$x \div x^3 = \frac{x}{x^3} = \frac{1}{x^2}$
6)	$2x^3$	$x^2 \cdot x^1 + x^3 = x^3 + x^3 = 2x^3$
7)	$-3x^3$	$4x^3 - 8x^3 + x^3 = -4x^3 + x^3 = -3x^3$
8)	$\frac{5x}{8}$	$\frac{x}{2} + \frac{x}{8} = \left(\frac{4}{4}\right)\frac{x}{2} + \frac{x}{8} = \frac{4x}{8} + \frac{x}{8} = \frac{4x+x}{8} = \frac{5x}{8}$
9)	$\frac{y^3}{2x}$	$\frac{x^2y^7}{2x^3y^4} = \frac{x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}{2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y} = \frac{y \cdot y \cdot y}{2 \cdot x} = \frac{y^3}{2x}$

2.2 Solving Linear Equations

The overall goal when solving equations is to isolate the variable on one side of the equation and send all of the numbers to the other side.

Rules

What you do to one side of the equation you <u>must</u> do to the other!

When you do something, it must be done to the <u>entire</u> side, not just one part.

To undo addition, subtract from both sides.

To undo subtraction, add to both sides.

To undo multiplication, divide on both sides.

To undo division, multiply on both sides.

Example

Consider the equation $3x + 7 = 1$.

To solve for x, begin by moving single integers to the right-hand side.

Subtract 7 from each side since subtraction is the opposite of addition.

$$\begin{array}{r} 3x + 7 = 1 \\ -7 \quad -7 \\ \hline 3x = -6 \end{array}$$

Now, divide the remaining number away from the variable.

$$\frac{3x}{3} = \frac{-6}{3}$$

Divide both sides by 3 since division is the opposite of multiplication.

$$x = -2$$

Final answer! Try plugging $x = -2$ back into the original equation!

Example

Consider the equation $2x - 3 - 2 = 3$.

To solve for x, begin by combining like terms.

$-3 - 2 = -5$

$$2x - 3 - 2 = 3$$
$$2x - 5 = 3$$

Now proceed by moving single integers to the right-hand side.

$$\begin{array}{r} 2x - 5 = 3 \\ +5 \quad +5 \\ \hline 2x = 8 \end{array}$$

Add 5 to both sides since addition is the opposite of subtraction (the negative 5).

Now, divide the remaining number away from the variable.

$$\frac{2x}{2} = \frac{8}{2}$$

Divide both sides by 2 since division is the opposite of multiplication.

$$x = 4$$

Final answer! Try plugging $x = 4$ back into the original equation!

Example

Consider the equation $-7x + 1 + 2x = 9x - 8 + 1$.

To solve for x, begin by combining like terms.

$$-7x + 1 + 2x = 9x - 8 + 1$$
$$-5x + 1 = 9x - 7$$

Now proceed by moving single integers to the right-hand side.

$$\begin{array}{rcl} -5x + 1 & = & 9x - 7 \\ -1 & & -1 \\ \hline -5x & = & 9x - 8 \end{array}$$

Make sure like terms are lined up!

If we want the variable to be isolated on one side, we must now move all variables to the left-hand side.

$$\begin{array}{rcl} -5x & = & 9x - 8 \\ -9x & & -9x \\ \hline -14x & = & -8 \end{array}$$

Now, divide the remaining number away from the variable.

Divide both sides by -14 since division is the opposite of multiplication.

$$\frac{-14x}{-14} = \frac{-8}{-14}$$

$$x = \frac{4}{7}$$

Final answer! Try plugging $x = \frac{4}{7}$ back into the original equation!

Practice Problems

Solve the following equations for the variable.

1) $2x - 3 = 1$

2) $4x + 6 = 7$

3) $14x - 5 = 9$

4) $7x + 8 = 1$

5) $2x + 3 = 7x - 1$

6) $5x + 2 = 3x - 6$

7) $x + 5 = x - 6$

8) $\frac{1}{2}x + \frac{3}{2} = 4$

9) $\frac{3}{4}x + 2 = x - 5$

10) $\frac{1}{2}x + \frac{3}{2}(x + 1) - \frac{1}{4} = 5$

11) $5x + 2 = \frac{1}{3}x$

12) $5(x + 2) = 3x$

13) $\frac{1}{5}x + \frac{1}{3} = 3$

14) $3x + \frac{1}{2}(x + 2) = -2$

Answers

1)	$x = 2$	$2x - 3 = 1 \to 2x = 4 \to x = 2$
2)	$x = \frac{1}{4}$	$4x + 6 = 7 \to 4x = 1 \to x = \frac{1}{4}$
3)	$x = 1$	$14x - 5 = 9 \to 14x = 14 \to x = 1$
4)	$x = -1$	$7x + 8 = 1 \to 7x = -7 \to x = -1$
5)	$x = \frac{4}{5}$	$2x + 3 = 7x - 1 \to 2x + 4 = 7x \to 4 = 5x \to \frac{4}{5} = x$
6)	$x = -4$	$5x + 2 = 3x - 6 \to 5x = 3x - 8 \to 2x = -8 \to x = -4$
7)	$no\ solution$	$x + 5 = x - 6 \to 5 = -6\ (but\ since\ 5 \neq -6, no\ solution)$
8)	$x = 5$	$\frac{1}{2}x + \frac{3}{2} = 4 \to \frac{1}{2}x = \frac{5}{2} \to x = 5$
9)	$x = 28$	$\frac{3}{4}x + 2 = x - 5 \to \frac{3}{4}x + 7 = x \to 7 = \frac{1}{4}x \to 28 = x$
10)	$x = \frac{15}{8}$	$\frac{1}{2}x + \frac{3}{2}(x + 1) - \frac{1}{4} = 5 \to \frac{1}{2}x + \frac{3}{2}x + \frac{3}{2} - \frac{1}{4} = 5 \to 2x + \frac{5}{4} = 5 \to 2x = \frac{15}{4} \to 2x = \frac{15}{4} \to x = \frac{15}{8}$
11)	$x = -\frac{3}{7}$	$5x + 2 = \frac{1}{3}x \to \frac{14}{3}x + 2 = 0 \to \frac{14}{3}x = -2 \to 14x = -6 \to x = -\frac{3}{7}$
12)	$x = -5$	$5(x + 2) = 3x \to 5x + 10 = 3x \to 10 = -2x \to -5 = x$
13)	$x = \frac{40}{3}$	$\frac{1}{5}x + \frac{1}{3} = 3 \to \frac{1}{5}x = \frac{8}{3} \to x = \frac{40}{3}$
14)	$x = -\frac{6}{7}$	$3x + \frac{1}{2}(x + 2) = -2 \to 3x + \frac{1}{2}x + 1 = -2 \to \frac{7}{2}x + 1 = -2 \to \frac{7}{2}x = -3 \to x = -\frac{6}{7}$

2.3 Solving Inequalities

Solving inequalities is almost exactly the same as solving equations! The equal sign $=$ has simply been replaced with one of the following: $< \ \leq \ > \ \geq$.

However, there is one extra rule to solving inequalities. If you ever **divide or multiply by a negative number** in your solving process, you must switch the direction the inequality sign is facing.

Example

Consider the equation $3x + 7 \geq 1$.

To solve for x, begin by moving single integers to the right-hand side.

Subtract 7 from each side since subtraction is the opposite of addition.

You do not have to switch the direction of the $\geq$ sign when subtracting. Only for multiplying or dividing by a negative number!

$$\begin{array}{r} 3x + 7 \geq 1 \\ -7 \quad\ \ -7 \\ \hline 3x \geq -6 \end{array}$$

Now, divide the remaining number away from the variable.

$$\frac{3x}{3} \geq \frac{-6}{3}$$

The $\geq$ sign does not switch directions since the 3 is positive.

$$x \geq -2$$

This is your final answer! It means that any number greater than or equal to -2 will satisfy the original inequality.

Try plugging $x = -2$ back into the original inequality! Notice that the inequality holds $(1 \geq 1)$.

Now try plugging in other numbers greater than -2! The inequality will hold!

Example

Consider the equation $-5x - 8 \geq 2$.

To solve for x, begin by moving single integers to the right-hand side.

Add 8 to both sides since addition is the opposite of subtraction.

$$\begin{array}{r} -5x - 8 \geq 2 \\ +8 \quad +8 \\ \hline -5x \geq 10 \end{array}$$

Now, divide the remaining number

$$\frac{-5x}{-5} \geq \frac{10}{-5}$$

The $\geq$ sign MUST change direction now, since we divided by a negative number!

$$x \leq -2$$

This is your final answer! It means that any number less than or equal to -2 will satisfy the original inequality.

Try plugging $x = -2$ back into the original inequality! Notice that the inequality holds ($2 \geq 2$).

Now try plugging in other numbers less than -2! The inequality will hold!

Example

If your final answer does has $>$ or $<$ involved (instead of $\geq$ or $\leq$), this **excludes** the number on the other side of the sign.

For example, $x > 5$ means the answer is any number greater than 5, but **not including 5**.

Example

Solve the following inequalities.

1) $x - 3 \geq 12$

2) $4s < -16$

3) $5w + 2 \leq -48$

4) $2k > 7$

5) $2y + 4 \leq y + 8$

6) $-3a \geq 9$

7) $-3(z - 6) > 2z - 2$

8) $-2n + 8 \leq 2n$

Answers

1)	$x \geq 15$	$x - 3 \geq 12 \rightarrow x \geq 15$
2)	$s < -4$	$4s < -16 \rightarrow s < -4$
3)	$w \leq -10$	$5w + 2 \leq -48 \rightarrow 5w \leq -50 \rightarrow w \leq -10$
4)	$k > \frac{7}{2}$	$2k > 7 \rightarrow k > \frac{7}{2}$
5)	$y \leq 4$	$2y + 4 \leq y + 8 \rightarrow 2y \leq y + 4 \rightarrow y \leq 4$
6)	$a \leq -3$	$-3a \geq 9 \rightarrow a \leq -3$
7)	$z < 4$	$-3(z - 6) > 2z - 2 \rightarrow -3z + 18 > 2z - 2 \rightarrow 18 > 5z - 2 \rightarrow 20 > 5z \rightarrow 4 > z$
8)	$n \geq 2$	$-2n + 8 \leq 2n \rightarrow 8 \leq 4n \rightarrow 2 \leq n$

2.4 Solving Systems of Equations

Sometimes there are two or more equations that are related because their variables are the same. This means that if you find that $x = 4$ in one equation, the x in the other equation is also guaranteed to be 4.

Most often you will have to create two equations from a word problem and then solve them as a system!

Example

The following is a system of equations:

$$2x + y = 5 \quad and \quad x - 2y = 15$$

The solution to this system is $x = 5$ and $y = -5$. Try plugging these values into each equation! They work for both!

Example

Consider the system: $2x + y = 5 \quad and \quad x - 2y = 15$

To solve this system, we must isolate one of the variables. In this case, it looks easy to isolate the x in the second equation, so we'll start with that. (You could just as easily choose to isolate the y variable in the first equation- it doesn't make a difference).

$$x - 2y = 15$$

$$x = 2y + 15$$

Use algebra here and isolate the variable. For more on solving equations, refer to Section 4.2.

(Continued on next page)

Example Continued

Notice, x and $2y + 15$ are exactly equal to one another. This means we can replace any x we see with $2y + 15$.

We will replace the x in the first equation with $2y + 15$.

$$2x + y = 5$$

$$2(2y + 15) + y = 5$$

$$4y + 30 + y = 5$$

$$5y + 30 = 5$$

Now that we have substituted that expression in for x, our first equation has only one variable to solve for! So now we solve for y.

$$5y + 30 = 5$$

$$5y = -25$$

$$y = -5$$

So $y = -5$! Now we can take that information and plug $y = -5$ in to either of our beginning equations in order to find x. It does not matter which equation we choose because they are both related!

$$x - 2y = 15$$

$$x - 2(-5) = 15$$

$$x + 10 = 15$$

$$x = 5$$

So our final answer is $x = 5$, $y = -5$.

Example

Solve the following systems of equations.

1) $2x + y = 5 \ \ and \ \ x - 2y = 15$

2) $y = 2x + 4 \ \ and \ \ y = -3x + 9$

3) $y = x + 3 \ \ and \ \ 42 = 4x + 2y$

4) $3x + y = 12 \ \ and \ \ y = -2x + 10$

5) $y = -3x + 5 \ \ and \ 5x - 4y = -3$

6) $y = -2 \ \ and \ \ 4x - 3y = 18$

7) $-7x - 2y = -13 \ \ and \ \ x - 2y = 11$

8) $6x - 3y = 5 \ \ and \ \ y - 2x = 8$

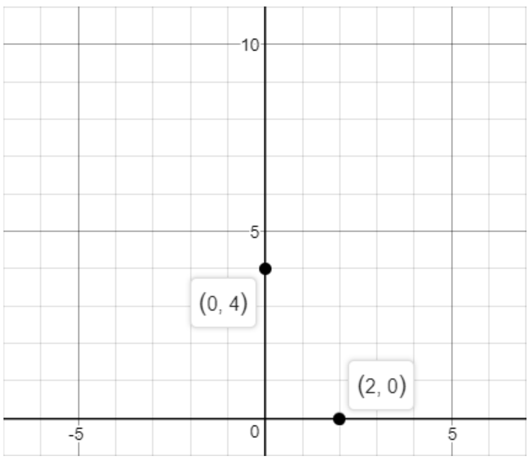
Answers

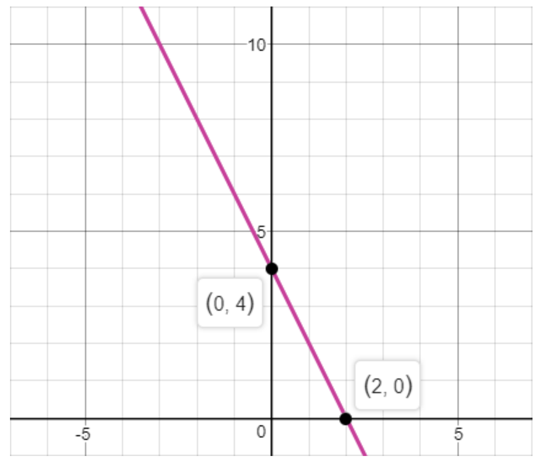
1)	$x = 5, y = -5$
2)	$x = 1, y = 6$
3)	$x = 6, y = 9$
4)	$x = \frac{2}{5}, y = \frac{54}{5}$
5)	$x = 1, y = 2$
6)	$x = 3, y = -2$
7)	$x = 3, y = -4$
8)	$no\ solution$

2.5 Graphing Linear Equations

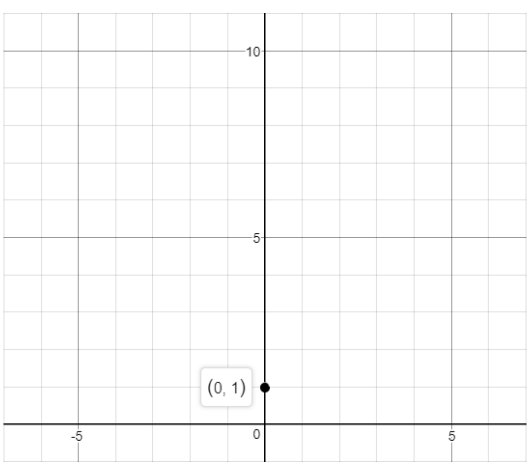
There are two methods to graph a line:

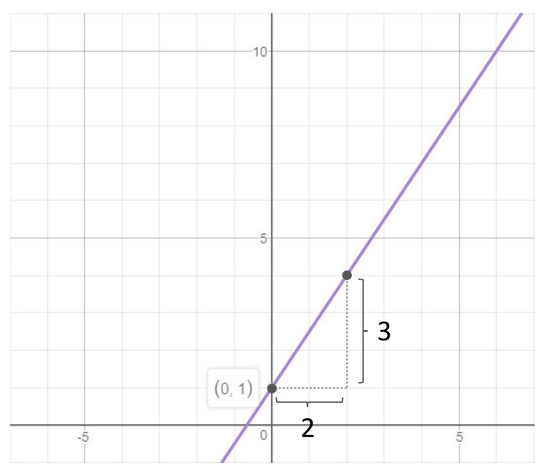
1) Use the x and y intercepts: If you have the coordinates of the x and y intercepts, plot those two points and connect them with a line.





2) Use slope-intercept form: If you have an equation in slope intercept form ($y = mx + b$), plot the y-intercept and then construct the line using the slope. If the equation is not in slope-intercept form, it can be rearranged to be in slope-intercept form.





$$m = \frac{3}{2}$$

Method 1

Consider the equation $6x + 2y = 12$.

Find the x-intercept:

To find the x-intercept, set y equal to zero and solve for x.

$$6x + 2y = 12$$

$$6x + 2(0) = 12$$

$$6x = 12$$

$$x = 2$$

This is the x-intercept, meaning the point $(2,0)$ is on the graph.

Find the y-intercept:

To find the y-intercept, set x equal to zero and solve for y.

$$6x + 2y = 12$$

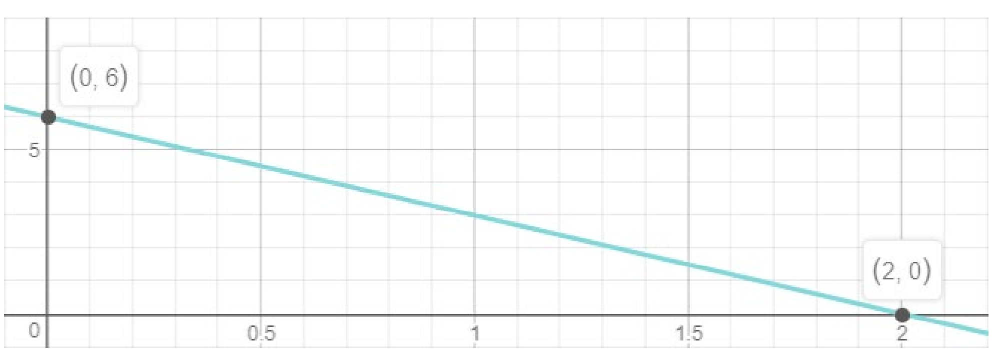
$$6(0) + 2y = 12$$

$$2y = 12$$

$$y = 6$$

This is the y-intercept, meaning the point $(0,6)$ is on the graph.

Now, plot the points $(2,0)$ and $(0,6)$ on the graph and connect them with a line:



Method 2

Consider the equation $6x + 2y = 12$.

First, use algebra to rearrange the equation:

$$6x + 2y = 12$$

$$2y = -6x + 12$$

$$y = -3x + 6$$

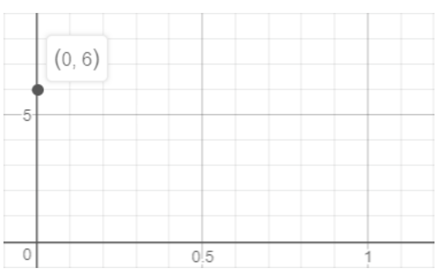
Now the equation is in y-intercept form ($y = mx + b$).

Then identify your slope and y-intercept:

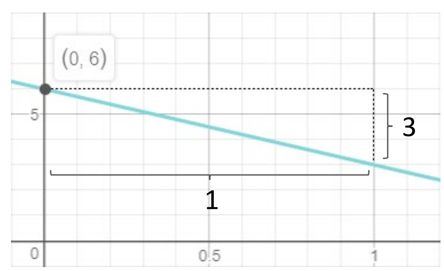
Compare $y = -3x + 6$ with $y = mx + b$.

In this case, $m = -3$ and $b = 6$. In other words, the slope is -3 and the y-intercept is at $(0,6)$.

Plot the y-intercept:



Use the slope to draw the rest of the line:

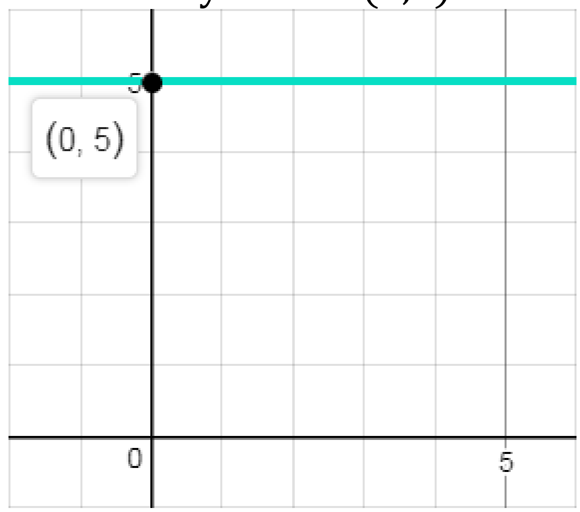
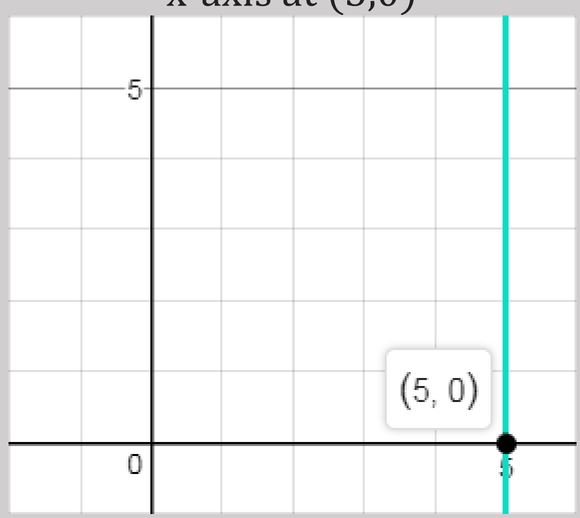


The slope is $-3 = \frac{-3}{1} = \frac{rise}{run}$

The slope is rewritten as a fraction to reveal the rise (numerator) and run (denominator). The rise is the amount the slope changes on the y-axis and the run is the amount the slope changes on the x-axis.

Graphs with one variable

If a graph only involves one variable, it can be graphed as a horizontal or vertical line. Below are two examples.

Equation Form	Resulting Graph
$y = 5$	A horizontal line that crosses the y-axis at (0,5) 
$x = 5$	A vertical line that crosses the x-axis at (5,0) 

Practice Problems

Graph the following equations.

1) $y = 4x + 1$

2) $y = 12x + 3$

3) $y = -3x + 2$

4) $y = -x + 4$

5) $y = 23x - 2$

6) $y = x$

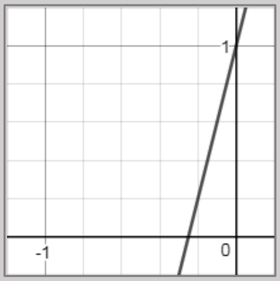
7) $3y - 12 = -6x$

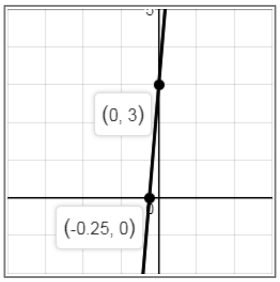
8) $x + 6y = 7$

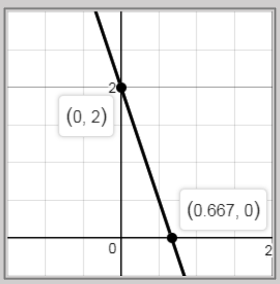
9) $x = 4$

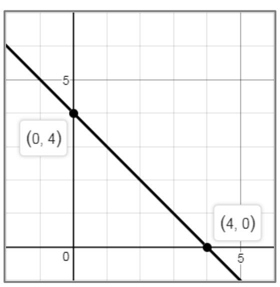
10) $y = -2.5$

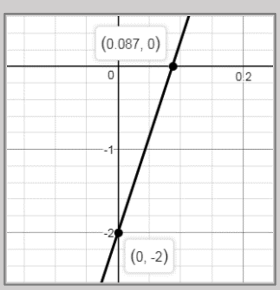
Answers

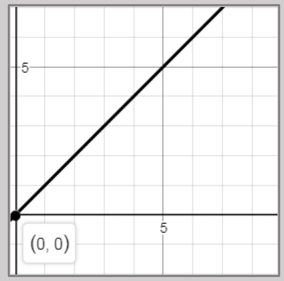
1) 

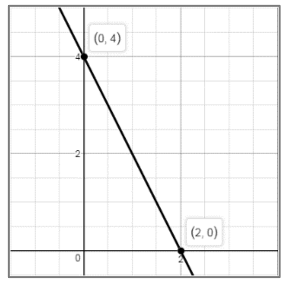
2) 

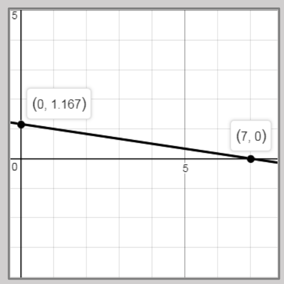
3) 

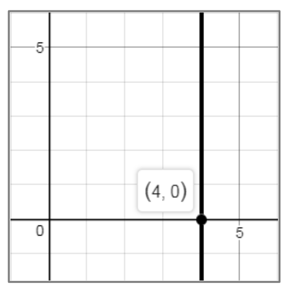
4) 

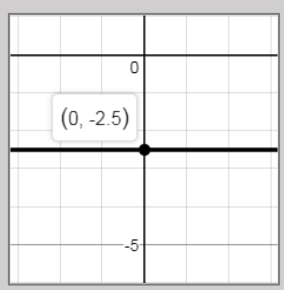
5) 

6) 

7) 

8) 

9) 

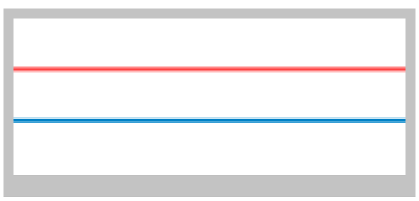
10) 

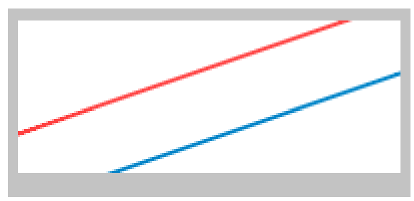
2.6 Building Lines

You may be asked to state the equation of a line based on the points it passes through or its slope. You may also be asked to write the equation of a line parallel or perpendicular to another line. If you don't know how to graph linear equations, see Section 2.5.

Parallel Lines

Lines are parallel if they have the same slope. Below are two examples of parallel lines. The red lines have different y-intercepts than the blue lines, but they have matching slopes. In order to construct parallel lines, just find two lines with the same slope.





Example

Find a line parallel to $y = 2x + 3$.

Any line that has the same slope ($m = 2$) is a correct answer! For instance $y = 2x + 4$ and $y = 2x - 24$.

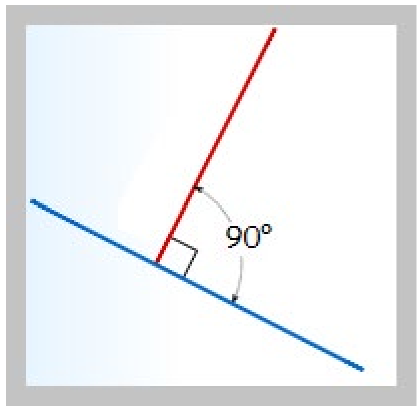
Example

Find a line parallel to $y = \frac{5}{7}x$.

Any line that has the same slope ($m = \frac{5}{7}$) is a correct answer! For instance $y = \frac{5}{7}x + 2$ and $y = \frac{5}{7}x - 10$.

Perpendicular Lines

Lines are perpendicular if they intersect at a 90° angle. Below is an example of a pair of perpendicular lines. The slopes of perpendicular lines are the negative reciprocals of one another.



Example

Find a line perpendicular to $y = 2x + 3$.

To find the perpendicular slope, take the negative reciprocal of $m = 2$. The reciprocal of 2 is $\frac{1}{2}$. Then make it negative. Therefore, any equation with the slope $m = -\frac{1}{2}$ is perpendicular to our original equation. For instance, $y = -\frac{1}{2}x + 3$ and $y = -\frac{1}{2}x - 1$.

Example

Find a line perpendicular to $y = -\frac{5}{4}x + 6$.

To find the perpendicular slope, take the negative reciprocal of $m = -\frac{5}{4}$. The reciprocal of $-\frac{5}{4}$ is $-\frac{4}{5}$. Then negate it, causing it to turn positive.

(Continued on the next page)

Example Continued

Therefore, any equation with the slope $m = \frac{4}{5}$ is perpendicular to our original equation. For instance, $y = \frac{4}{5}x + \frac{1}{2}$ and $y = \frac{4}{5}x - 3$.

Line Equations

When given characteristics of a line, you can build the corresponding equation. When given a y-intercept and a slope, use the equation $\boldsymbol{y = mx + b}$. Plug in your slope for m and your y-intercept for b.

When given a point (x_1, y_1) and a slope m, use the equation $\boldsymbol{y - y_1 = m(x - x_1)}$. Then just algebraically rearrange your result into the form $y = mx + b$.

When given only two points, (x_1, y_1) and (x_2, y_2), first plug these points into the slope formula $\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$ to find your m. Then just use $y - y_1 = m(x - x_1)$ like in the previous case.

Example

Find the equation of the line which passes through the points $(1,2)$ and $(4,9)$.

First we find the slope using $m = \frac{y_2 - y_1}{x_2 - x_1}$. Label the points (x_1, y_1) and (x_2, y_2). In this case $x_1 = 1$, $y_1 = 2$, $x_2 = 4$, and $y_2 = 9$. So $m = \frac{9-2}{4-1} = \frac{7}{3}$. Now choose either point, and plug it in to $y - y_1 = m(x - x_1)$. We'll use the point $(1,2)$ here. $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{7}{3}(x - 1)$$

$$y - 2 = \frac{7}{3}x - \frac{7}{3}$$

$$y = \frac{7}{3}x - \frac{1}{3}$$

Therefore the equation of the line that passes through the desired points is $y = \frac{7}{3}x - \frac{1}{3}$.

Practice Problems

1) Find the equation of a line parallel to $y = 3x + 7$.
2) Find the equation of a line parallel to $y = \frac{1}{2}x - 2$.
3) Find the equation of a line parallel to $y = -\frac{3}{2}x + 9$.
4) Find the equation of a line perpendicular to $y = \frac{1}{2}x + 1$.
5) Find the equation of a line perpendicular to $y = 5x + 12$.
6) Find the equation of a line perpendicular to $y = -\frac{2}{7}x - 8$.
7) Find the equation of the line that has a slope of 2 and intercepts the y-axis at $y = 7$.
8) Find the equation of the line that has a slope of $\frac{4}{5}$ and intercepts the y-axis at $y = -3$.
9) Find the equation of the line which passes through the points $(4,3)$ and $(2,2)$.
10) Find the equation of the line which passes through the points $(-4,5)$ and $(0,0)$.

Answers

1)	$y = 3x \pm anything$
2)	$y = \frac{1}{2}x \pm anything$
3)	$y = -\frac{3}{2}x \pm anything$
4)	$y = -2x \pm anything$
5)	$y = -\frac{1}{5}x \pm anything$
6)	$y = \frac{7}{2}x \pm anything$
7)	$y = 2x+7$
8)	$y = \frac{4}{5}x - 3$
9)	$y = \frac{1}{2}x + 1$
10)	$y = -\frac{5}{4}x$

3

Logarithms

3.1 Exponential Form vs. Logarithmic Form

Logarithms (or *logs* for short) are the inverse of exponentiation. Consider the following example:

$$8 = 2^3$$

This equation shows that "8 is 2 to the power of 3." Logs take the left side of the equation and the base of the exponent to identify the power. So in terms of logs, this equation also shows that "log base 2 of 8 is 3." This is written as follows:

$$\log_2 8 = 3$$

This relationship is useful for solving equations with a variable in the exponent, such as:

$$10^x = 100$$
$$\leftrightarrow$$
$$\log_{10} 100 = x$$
$$\leftrightarrow$$
$$2 = x$$

Many logs can be evaluated by hand, but most are evaluated using a calculator. Techniques for evaluating logarithms will be covered in Section 3.2. For this section the focus is changing an equation from one form to the other. The general formula is:

$$b^c = a \longleftarrow \text{Exponential Form}$$
$$\leftrightarrow$$
$$\text{Logarithmic Form} \longrightarrow \log_b a = c$$

Equations of these forms can always be interchanged.

Example

Write the exponential equation $2^5 = 32$ in logarithmic form.

Original equation in exponential form	$2^5 = 32$
Identify a, b, and c in the exponential equation	$\underset{b}{2}^{\underset{c}{5}} = \underset{a}{32}$
Rearrange into logarithmic form (final answer)	$\log_2 32 = 5$

Example

Write the logarithmic equation $\log_9 81 = 2$ in exponential form.

Original equation in logarithmic form	$\log_9 81 = 2$
Identify a, b, and c in the logarithmic equation	$\log_{\underset{b}{9}} \underset{a}{81} = \underset{c}{2}$
Rearrange into exponential form (final answer)	$9^2 = 81$

The next few sections cover more properties of logarithms.

Practice Problems

Write the following exponential equations as logarithmic equations.

1) $10^2 = 100$
2) $10^3 = 1000$
3) $5^4 = 625$
4) $2^1 = 2$
5) $2^6 = 64$
6) $x^2 = 10$
7) $w^y = z$

Write the following logarithmic equations as exponential equations.

8) $\log_7 49 = 2$
9) $\log_2 8 = 3$
10) $\log_5 5 = 1$
11) $\log_4 1 = 0$
12) $\log_3 27 = 3$
13) $\log_{10} 1000 = x$
14) $\log_x y = z$

Answers

1)	$\log_{10} 100 = 2$
2)	$\log_{10} 1000 = 3$
3)	$\log_5 625 = 4$
4)	$\log_2 2 = 1$
5)	$\log_2 64 = 6$
6)	$\log_x 10 = 2$
7)	$\log_w z = y$
8)	$7^2 = 49$
9)	$2^3 = 8$
10)	$5^1 = 5$
11)	$4^0 = 1$
12)	$3^3 = 27$
13)	$10^x = 1000$
14)	$x^z = y$

3.2 Rules of Logarithms

Logarithms are derived from exponents, so all logarithmic rules are based on the rules of exponents.

Rules

$\mathbf{\log_b(xy) = \log_b(x) + \log_b(y)}$	$\log_2(15) = \log_2(3 \cdot 5) = \log_2(3) + \log_2(5)$
$\mathbf{\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)}$	$\log_2(5) = \log_2\left(\frac{10}{2}\right) = \log_2(10) - \log_2(2)$
$\mathbf{\log_b(x^n) = n \cdot \log_b(x)}$	$\log_2(9) = \log_2(3^2) = 2\log_2(3)$
$\mathbf{\log_b(a) = \frac{\log_{10}(a)}{\log_{10}(b)}}$ Change of base formula	$\log_3(5) = \frac{\log_{10}(5)}{\log_{10}(3)}$ $(This\ formula\ is\ useful\ when\ using\ a\ calculator)$
$\mathbf{\log_b(b) = 1}$	$\log_6(6) = 1$
$\mathbf{\log_b(1) = 0}$	$\log_4(1) = 0$

Notation

$\mathbf{\log_{10}(x) = \log x}$	$Since\ a\ base\ of\ 10\ is\ very\ common,$ $\log_{10}\ is\ often\ written\ as\ just\ \log$
$\mathbf{\log_e(x) = \ln x}$	$The\ number\ e\ (\approx 2.71828)\ is\ also\ a\ very\ common\ base, so\ \log_e\ has\ its\ own\ notation\ as\ \ln$

Note On Negatives

For $\log_b(x)$, x cannot be negative. For example, $\log_{10}(-4)$ has no solution.

Example

Expand the logarithmic expression $\log_5 xy^2$.

Original expression	$\log_5 xy^2$
Use multiplication rule	$\log_5 x + \log_5 y^2$
Use power rule	$\log_5 x + 2\log_5 y$ (logarithm is now fully expanded)

Example

Condense the logarithmic expression $\log_2 x - \log_2 y + \log_2 z$.

Original expression	$\log_2 x - \log_2 y + \log_2 z$
Rearrange	$\log_2 x + \log_2 z - \log_2 y$
Use multiplication rule	$\log_2 xz - \log_2 y$
Use division rule	$\log_2 \frac{xz}{y}$ (logarithm is now fully condensed)

Example

Condense the logarithmic expression $3\log_{11} q + 2\log_{11} r + \frac{1}{2}\log_{11} s - \log_{11} t$.

Original expression	$3\log_{11} q + 2\log_{11} r + \frac{1}{2}\log_{11} s - \log_{11} t$
Use power rule	$\log_{11} q^3 + \log_{11} r^2 + \log_{11} s^{\frac{1}{2}} - \log_{11} t$
Rewrite fractional power as root	$\log_{11} q^3 + \log_{11} r^2 + \log_{11} \sqrt{s} - \log_{11} t$
Use multiplication rule	$\log_{11} q^3 r^2 \sqrt{s} - \log_{11} t$
Use division rule	$\log_{11} \frac{q^3 r^2 \sqrt{s}}{t}$ (logarithm is now fully condensed)

Example

Expand the logarithmic expression $\log_8 \frac{\sqrt[3]{d} \cdot c}{h}$.

Original expression	$\log_8 \frac{\sqrt[3]{d} \cdot c}{h}$
Use multiplication/division rules	$\log_8 \sqrt[3]{d} + \log_8 c - \log_8 h$
Rewrite root as fractional power	$\log_8 d^{\frac{1}{3}} + \log_8 c - \log_8 h$
Use power rule	$\frac{1}{3}\log_8 d + \log_8 c - \log_8 h$ (logarithm is now fully expanded)

Example

Evaluate the logarithmic expression $\log_2 8$. Simplify down to a single number.

Original expression	$\log_2 8$
Rewrite 8 as 2^3	$\log_2(2^3)$
Use power rule	$3\log_2 2$
$\log_2 2 = 1$	$3 \cdot 1$
Final answer	3

Example

Evaluate the logarithmic expression $\ln e$. Simplify down to a single number.

Original expression	$\ln e$
Rewrite $\ln e$ as $\log_e e$	$\log_e e$
$\log_e e = 1$, final answer	1

Example

Evaluate the logarithmic expression $\log_{10} 1$. Simplify down to a single number.

Original expression	$\log_{10} 1$
Rewrite 1 as 10^0	$\log_{10} 10^0$
Use power rule	$0 \cdot \log_{10} 10$
$\log_{10} 10 = 1$	$0 \cdot 1$
Final answer	0

Practice Problems

Expand the logarithmic expressions.

1) $\log_2 \frac{q \cdot r}{s}$
2) $\log_{10} \frac{10}{x^2}$
3) $\ln(x^3 y)$
4) $\log \frac{7a}{b}$

Condense the logarithmic expressions.

5) $5\log_9 x + \log_9 y$
6) $\log a + \log b + 2\log c - \log d$
7) $-\log_{11} w + 2\log_{11} x - \log_{11} y + \frac{1}{2}\log_{11} z$
8) $\ln x + \ln y - 2\ln x$

Simplify the logarithmic expression down to a single number.

9) $3\log_4 2 - \log_4 8$
10) $\log_3 3 + \log_8 8 + \log 10$
11) $\ln e^6 + \ln e$
12) $\log 100$
13) $\log_{15}(-3)$
14) $\ln 2 + \ln 6 - \ln 12$

Use the change of base formula to rewrite the expression (don't simplify).

15) $\log_7 5$
16) $\log_2 \pi$

Answers

1)	$\log_2 q + \log_2 r - \log_2 s$
2)	$\log_{10} 10 - 2\log_{10} x$
3)	$3\log_e x + \log_e y$
4)	$\log 7 + \log a - \log b$
5)	$\log_9 (x^5 y)$
6)	$\log \frac{abc^2}{d}$
7)	$\log_{11} \frac{x^2\sqrt{z}}{wy}$
8)	$\ln \frac{y}{x}$
9)	0
10)	3
11)	7
12)	2
13)	$No\ solution$
14)	0
15)	$\frac{\log_{10}(5)}{\log_{10}(7)}$
16)	$\frac{\log_{10}(\pi)}{\log_{10}(2)}$

3.3 Solving Logarithmic Equations

Using the rules of logarithms and the ability to rearrange logarithmic and exponential equations, you can solve for variables that might not have been easy to solve for before.

Practice Problems

Solve the following logarithmic expressions.

1) $\log_x 144 = 2$
2) $\log_x 5 = \frac{1}{4}$
3) $\log_9 x = 2$
4) $\log_4 x = 3$
5) $\log_{25} x = -\frac{1}{2}$
6) $\log_{\frac{1}{10}} x = -2$
7) $\log_x 49 = 2$
8) $\log_x 5 = \frac{1}{3}$
9) $\log_x 32 = \frac{5}{2}$
10) $\log_2 x = 3$
11) $\log_{10} x = 3$
12) $\log_{16} x = 1$
13) $3 \log_x 3 = 1$
14) $4 \log_x 32 = 5$

Answers

1)	$x = 12$
2)	$x = 625$
3)	$x = 81$
4)	$x = 64$
5)	$x = \frac{1}{5}$
6)	$x = 100$
7)	$x = 7$
8)	$x = 125$
9)	$x = 4$
10)	$x = 8$
11)	$x = 1000$
12)	$x = 16$
13)	$x = 27$
14)	$x = 16$

3.4 Common Base Technique

When two exponential expressions with the same base are equal to one another, their exponents can also be set equal to one another. Consider the following example:

$$2^x = 2^5$$

Notice how the only possible value for x is 5. Setting the exponents equal to one another, we get $x = 5$. We are only allowed to set the exponents equal to each other when the bases are the same (in this case, the base on each side of the equation was 2).

Now consider another example:

$$5^{2x-3} = 5^{x+5}$$

Since the bases are the same (5), we can set the exponents equal to one another:

$$2x - 3 = x + 5$$
$$\leftrightarrow$$
$$x = 8$$

Practice Problems

Solve the equations for x:

1) $3^{10} = 3^x$
2) $11^{6x} = 11^{36}$
3) $2^{2x+9} = 2^{3x-9}$
4) $y^5 = y^{2x}y^1$
5) $\frac{9^1}{9^x} = 9^x$
6) $\frac{\left(a^{\frac{1}{3}}\right)^9}{a^{-1}} = \frac{a^{4x}}{a^{12}}$
7) $7^{11x-5} = \frac{49^x}{7^4}$

Answers

1)	$x = 10$
2)	$x = 6$
3)	$x = 18$
4)	$x = 2$
5)	$x = \frac{1}{2}$
6)	$x = 4$
7)	$x = \frac{1}{9}$

3.5 Exponential Models

Word problems modelling growth or decay using exponents may be on the test. Logarithms can be used to solve these equations

Investment Growth

We can use logarithms to determine the growth and decay of certain models. Investment growth is a perfect example when dealing with exponential growth.

Let's say that $\$1000$ is deposited in an account that pays 10% interest annually, compounded monthly. This means that the interest rate each month is

$$\frac{10\%}{12\ months} = .83\%\ interest\ per\ month$$

At the end of the month, $.83\%$ of the amount deposited in the account is added to the account. Let's look at some conditions we have specified:

Initial amount: $\$1000$

Time period: 1 month

Monthly growth rate: $.83\%$

Monthly growth factor: $1 + .83 = 1.83$

For the monthly growth factor, 1 is given by the total amount in the account added by the percent of that total ($83\% = .83$).

The amount of money in the account at any given month x can be modeled by the exponential function

$$A(x) = 1000(1.83)^x$$

In general, if the annual interest rate r expressed as a decimal, is compounded n amount of times each year then in each period the interest rate is $r \div n$. In t years, there are nt time periods with an initial value of P deposited into the account. This leads to the formula:

$$\text{Compound Interest: } A(t) = P(1 + \tfrac{r}{n})^{nt}$$

Lets look at our values again:

$P = \$1000$

$r = .83$ (decimal percent)

$n = 1\ month$

If we substitute these values into the general compound interest formula we get:

$$A(t) = 1000(1.83)^t$$

This equation is identical to the first one we derived with a replacement of t for x.

If money is invested and compounded annually, use the following formula:

$$A(t) = P(1 + r)^t$$

Example

Majid invests $\$5000$ in a high yield bond that pays 12% annual interest, compounded every 2 months.

1) Find a formula for the amount of money in the account after t amount of years.
2) What is the amount of money in the account after 5 years?
3) How long will it take for Majid's investment to grow to \$15,000?

Solution:

1) We use the compound interest formula and plug in our new conditions:

   Initial value: $P = \$5000$

   Interest rate: $r = 12\% = .12$ as a decimal

   Compound monthly: $n = 2$ months

$$A(t) = P(1 + \frac{r}{n})^{nt}$$

$$A(t) = 5000(1 + \frac{.12}{2})^{2t}$$

$$A(t) = 5000(1.06)^{2t}$$

2) To find the amount in the account after 5 years we replace t with 5 in the formula we just derived

$$A(t) = 5000(1.06)^{2(5)}$$

$$A(t) = \$8954$$

So in 5 years the balance in the account will sit at $8954.

3) We want to know what time (or value of t) the amount of money will reach a value of $15,000. So we need to find the value t.

$$A(t) = 5000(1.06)^{2t}$$

$$15{,}000 = 5000(1.06)^{2t}$$

$$3 = 1.06^{2t}$$

$$\log(3) = \log((1.06)^{2t})$$

$$\log(3) = (2t)\log(1.06)$$

$$t = \frac{\log(3)}{2\log(1.06)}$$

$$t = 9.42\ years$$

Decay

Sometimes you'll use the formula for exponential decay (using the same variables as before):

$$A(t) = P(1-r)^t$$

A great example of modeling decay is with radioactive material. The amount of radioactive material present at time t is given by:

$$R(t) = R_0 e^{kt} \qquad k < 0$$

$$R_0\ is\ the\ original\ amount\ of\ material\ and\ k\ represents\ the\ decay\ rate$$

Example

The half-life of Carbon-12 is 5730 years. How much Carbon-12 is left in an object that has been decaying for 3750 years if the original amount of Carbon-12 is $30{,}000$ atoms?

The half-life of an atom is the time it takes for half of the amount of an atom to decay. Using this value for t we can solve for k and then use that value along with other initial conditions to find the amount of atoms left after $t = 3750$ years. First, solve for k:

If we start with 2 atoms of carbon-12, then we can see that after 5730 years, half of 2 would be our current amount. That's why we're plugging in 2 and 1. We could use any initial value and come to the same result.

$$R(t) = R_0 e^{kt}$$

$$1 = 2e^{5730k}$$

$$\frac{1}{2} = e^{5730k}$$

$$\ln\left(\frac{1}{2}\right) = 5730k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{5730}$$

$$k = -.00012$$

We now use our k value to find the amount of atoms left after 3750 years if we initially start with $30{,}000$ atoms.

$$R(t) = R_0 e^{kt}$$

$$R(3750) = 30{,}000e^{-.00012(3750)}$$

$$R = 30{,}000e^{-.45363}$$

$$R = 30{,}000(.6353)$$

$$R = 19{,}059$$

We conclude that there are $19{,}059$ atoms of Carbon-12 after 3750 years of decay.

Population Growth

We can also see models grow exponentially. Let's take the example of bacteria. In an idealized environment, the amount of bacteria can be modeled by the equation:

$$N(t) = Ne^{kt}$$

N represents the initial amount of bacteria present and k is the growth factor.

Example

Suppose that at the start of an experiment in a lab, 1200 bacteria are introduced into an environment. After some time, a scientist found the amount of bacteria to be 1500. Knowing that the growth rate is 14.46 for this bacteria, what amount of time had passed (in hours)?

Given our initial conditions of

$N(t) = 1500$

$N = 1200$

$k = 14.46$

Let's solve for t:

$$N(t) = Ne^{kt}$$

$$1500 = 1200e^{14.46t}$$

$$\frac{1500}{1200} = e^{14.46t}$$

$$\ln\left(\frac{15}{12}\right) = 14.46t$$

$$t = \frac{\ln(1.25)}{14.46}$$

$$t = .015\ hours$$

Practice Problems

Set up the following equations. You do not need to solve them.

1) Sam invests $\$4025$ in a high yield bond that pays 5% annual interest, compounded every 2 months.
2) Suppose that at the start of an experiment in a lab, 7700 bacteria are introduced into an environment. After some time, a scientist found the amount of bacteria to be 8000. The growth rate is 42.30 for this bacteria.
3) $\$10{,}000$ is invested at 7% interest compounded annually.
4) The value of a new $\$40{,}000$ car decreases by 15% per year.

Answers

1)	$A(t) = 4025(0.025)^{2t}$
2)	$8000 = 7700e^{42.30t}$
3)	$A(t) = 10{,}000(1.07)^{t}$
4)	$A(t) = 40{,}000(0.85)^{t}$

4

Functions

4.1 Introduction, Domain, and Range

A function is a relation from a set of inputs to a set of possible outputs where each input is related to exactly one output. Think of it as taking one number as an initial condition and producing another number as a result. You might feel that a lot of the following information doesn't seem new. Functions are very similar to equations, but they have slightly different notation and some stricter rules. Functions always have one output for every input.

Example

Consider the equation $y = x + 1$. Here, x is an *independent* variable. If we choose a number to replace x, we get a corresponding y value. So we call the y variable *dependent*. The value of y will depend on what you choose for your value of x.

The relationship between input and output can be written as an ordered pair (x, y):

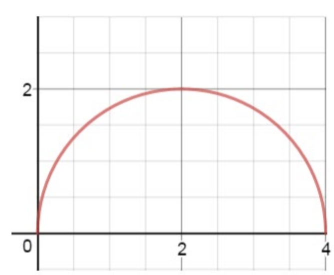
Ordered pairs
(0,1)
(1,2)
(2,3)

$y = x + 1 = ?$	Input	Output
$y = 0 + 1 = 1$	$x = 0$	$y = 1$
$y = 1 + 1 = 2$	$x = 1$	$y = 2$
$y = 2 + 1 = 3$	$x = 2$	$y = 3$

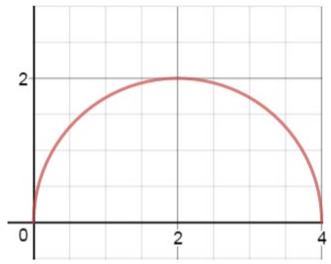
Domain and Range

The *domain* of a function is the set of values for the independent variable. These input values for x must produce a real number for the output of y. Sometimes the domain is specified explicitly and other times you have to find the domain from the given function.
The *range* of a function is the set of y values- the output that is produced by a domain. Ask yourself what the smallest and largest values of y are that will be produced from all the values of x.

The function on the right has x-values between $x = 0$ and $x = 4$ and y-values between $y = 0$ and $y = 2$. So the domain of the function is 0 to 4 and the range is 0 to 2. The notation for domain and range will be explained on the next page.



Notation for Domain/Range



Set notation is used for domain and range. Parentheses are used to note a set of values in this case. Since the graph to the left uses every single x value between 0 and 4 (including 0 and 4), the notation would be $[0,4]$. This notation says that every value between 0 and 4 is used, including 0 and 4. If the domain did not include 0 and 4, the notation would be $(0,4)$. Similarly, the range for this graph is $[0,2]$.

Example

What is the domain of $y = 2x + 3$?
(Think of all the values for x that would result in a real number y)

Thinking through it (or looking at a graph), we can determine that all real numbers substituted in for x will produce a real number for y. Therefore the domain of the function $y = 2x + 3$ is $(-\infty, \infty)$. In this case, *all* numbers work, so the set of possible values stretches from negative infinity to infinity. Whenever an infinity sign is used, these parentheses ( ) are used instead of [ ]. This is because infinity is a concept, not a number- so it cannot be included in the set of values.

All linear equations (equations of the form $y = mx + b$) have a domain and range of $(-\infty, \infty)$.

Example

What is the domain of $y = \sqrt{x + 15}$?
(Think of all the values for x that would result in a real number y)

Some values of x would result in an imaginary number (a negative number under the square root, which is not allowed) for y, so those x-values cannot be included in the domain. For example, $x = -16$ would produce the y-value $y = \sqrt{-1}$ which is not a real number, so -16 is not in the domain. The square root of any number less than 0 does not exist. Therefore any value for x that will produce a negative output under the radical will not be part of the domain for this function.

(Continued on the next page)

Example Continued

So we need to think of the smallest possible x-value that produces a zero under the square root. We could solve for this by equating $0 = x + 15$ and solving for x:

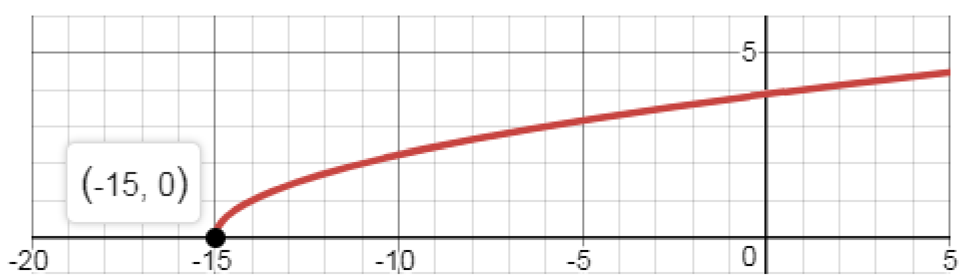
$$x + 15 = 0$$

$$\leftrightarrow$$

$$x = -15$$

If we plug -15 in for x, the value of $y = \sqrt{-15 + 15} = \sqrt{0} = 0$.
Thus -15 is the smallest value of x that will produce a real number for y.
We can also see that any positive value for x will yield a real number for y;
Therefore the domain of $y = \sqrt{x + 15}$ is $[-15, \infty)$.

This is reflected on the graph:



Example

Let's look at the Domain for the two examples:

$$y = 2x + 3 \qquad \text{D: } (-\infty, \infty)$$
$$y = \sqrt{x + 15} \qquad \text{D: } [-15, \infty)$$

The ranges for each equation are not bounded as the x-values increase towards ∞. So the range for each will approach ∞. For the first equation, the range will also approach $-\infty$ as the x-values go toward $-\infty$. But for the second equation, there is no way for y to be a negative number because of the radical, so the range is positive. The resulting ranges are:

$$y = 2x + 3 \qquad \text{R: } (-\infty, \infty)$$
$$y = \sqrt{x + 15} \qquad \text{R: } [0, \infty)$$

Example

What is the domain of $y = \frac{1}{x-2}$?

(Think of all the values for x that would result in a real number y)

In the case of fractions, the most important rule to remember is that dividing by zero is not allowed. So whatever values of x make the denominator equal to zero must be excluded from the domain. In this case, we can see that $x = 2$ would cause the denominator to become zero.

$$x - 2$$

$$\leftrightarrow$$

$$2 - 2$$

$$\leftrightarrow$$

$$0$$

So the domain is all real numbers except 2.

This domain is notated as $(-\infty, 2) \cup (2, \infty)$. The "$\cup$" symbol in between stands for "union." This means we are uniting the set of values $(-\infty, 2)$ and the set of values $(2, \infty)$ into one large set of values. Since the open brackets ( ) are used around the 2, that value is not included. It is interpreted as "all values from negative infinity up to two, and all values after two to infinity," which was what we concluded from finding the domain.

On the test, you are more likely to see domain written in a form such as "all real numbers except for 2," which is the same as $(-\infty, 2) \cup (2, \infty)$.

Practice Problems

Find the domain and range for 1-5. Then just find the domain for 6-10.

1) $y = 5x + 2$

2) $y = \frac{1}{2}x - 6$

3) $y = 7x$

4) $y = \sqrt{x}$

5) $y = \sqrt{x - 3}$

6) $y = \frac{1}{x-4}$

7) $y = \frac{1}{x}$

8) $y = \frac{x+3}{x+7}$

9) $y = x^2$

10) $y = |x|$

Answers

1)	$D: (-\infty, \infty)$ $R: (-\infty, \infty)$	
2)	$D: (-\infty, \infty)$ $R: (-\infty, \infty)$	
3)	$D: (-\infty, \infty)$ $R: (-\infty, \infty)$	
4)	$D: [0, \infty)$ $R: [0, \infty)$	
5)	$D: [3, \infty)$ $R: [0, \infty)$	
6)	$D: (-\infty, 4) \cup (4, \infty)$	or all real numbers except for 4
7)	$D: (-\infty, 0) \cup (0, \infty)$	or all real numbers except for 0
8)	$D: (-\infty, -7) \cup (-7, \infty)$	or all real numbers except for -7
9)	$D: (-\infty, \infty)$	or all real numbers
10)	$D: (-\infty, \infty)$	or all real numbers

4.2 Evaluating Functions

Evaluating functions is just like substitution. First, function notation will be introduced.

Function Notation

Instead of writing a function as $y = something$, you will see a function expressed as $f(x) = something$. This is read as "f of x." Other letters can be used other than f in order to distinguish different functions. This notation is meant to indicate that the function output is dependent upon the variable x.

Here are two examples of function notation:

$$f(x) = 2x + 3$$
$$g(x) = \sqrt{x + 15}$$

We can also refer to these functions just by their letter name. The function f is the top one, g is the bottom one.

Evaluating Functions

To evaluate a function, substitute the input for the function's variable. Consider the function $f(x) = 2x + 3$. Below, f is evaluated at x, 0, 1, 2, and -3.

$$f(x) = 2x + 3$$
$$f(0) = 2(0) + 3 = 3$$
$$f(1) = 2(1) + 3 = 5$$
$$f(2) = 2(2) + 3 = 7$$
$$f(-3) = 2(-3) + 3 = -3$$

In all of the functions we have seen, the variable has been x and the input has been a number substituted in place of x. This is not always the case. A function can be evaluated with any other variable or expression as input.

Examples of this are on the next page.

Evaluating Functions Continued

Below f is evaluated at y, $x+5$, x^2, and $x+h$.

$$f(y) = 2y + 3$$
$$f(x+5) = 2(x+5) + 3 = 2x + 13$$
$$f(x^2) = 2(x^2) + 3 = 2x^2 + 3$$
$$f(x+h) = 2(x+h) + 3 = 2x + 2h + 3$$

Example

Find $g(14)$ for the function $g(w) = w(w - 16)$.

$$g(14) = 14(14 - 16)$$
$$g(14) = 14(-2)$$
$$g(14) = -28$$

Example

Find $f(5)$ for the function $f(x) = 3x + 12$.

$$f(5) = 3(5) + 12$$
$$f(5) = 15 + 12$$
$$f(5) = 27$$

Example

Find $h(2w)$ for the function $h(x) = x^2$.

$$f(2w) = (2w)^2$$
$$f(2w) = 4w^2$$

Practice Problems

Evaluate the following functions.

1) Find $f(2)$ for the function $f(x) = x^2$.

2) Find $f(-5)$ for the function $f(x) = x^2$.

3) Find $g\left(\frac{1}{2}\right)$ for the function $g(x) = 2x + 4$.

4) Find $g(-2)$ for the function $g(x) = 2x + 4$.

5) Find $h(-5)$ for the function $h(x) = \sqrt{x + 9}$.

6) Find $d(3)$ for the function $d(x) = \frac{x+2}{x-2}$.

7) Find $g(t)$ for the function $g(x) = 2x + 4$.

8) Find $g(2x)$ for the function $g(x) = 2x + 4$.

9) Find $f(2x)$ for the function $f(x) = x^2$.

10) Find $f(x^2)$ for the function $f(x) = x^2$.

11) Find $f\left(\sqrt{s}\right)$ for the function $f(x) = x^2$.

12) Find $h(x - 3)$ for the function $h(x) = \sqrt{x + 9}$.

13) Find $k(r^2 + 1)$ for the function $k(x) = \sqrt{9 - x}$.

14) Find $d\left(\sqrt{x}\right)$ for the function $d(x) = \frac{x+2}{x-2}$.

15) Find $d(x + 1)$ for the function $d(x) = \frac{x+2}{x-2}$.

Answers

1)	$f(2) = 4$	$x^2 = (2)^2 = 4$
2)	$f(-5) = 25$	$x^2 = (-5)^2 = 25$
3)	$g\left(\frac{1}{2}\right) = 5$	$2x + 4 = 2\left(\frac{1}{2}\right) + 4 = 1 + 4 = 5$
4)	$g(-2) = 0$	$2x + 4 = 2(-2) + 4 = -4 + 4 = 0$
5)	$h(-5) = 2$	$\sqrt{x+9} = \sqrt{(-5)+9} = \sqrt{4} = 2$
6)	$d(3) = 5$	$\frac{x+2}{x-2} = \frac{(3)+2}{(3)-2} = \frac{5}{1} = 5$
7)	$g(t) = 2t + 4$	$2x + 4 = 2(t) + 4 = 2t + 4$
8)	$g(2x) = 4x + 4$	$2x + 4 = 2(2x) + 4 = 4x + 4$
9)	$f(2x) = 4x^2$	$x^2 = (2x)^2 = 4x^2$
10)	$f(x^2) = x^4$	$x^2 = (x^2)^2 = x^4$
11)	$f\left(\sqrt{s}\right) = s$	$x^2 = \left(\sqrt{s}\right)^2 = s$
12)	$h(x-3) = \sqrt{x+6}$	$\sqrt{x+9} = \sqrt{(x-3)+9} = \sqrt{x+6}$
13)	$k(r^2+1) = \sqrt{8-r^2}$	$\sqrt{9-x} = \sqrt{9-(r^2+1)} = \sqrt{9-r^2-1} = \sqrt{8-r^2}$
14)	$d\left(\sqrt{x}\right) = \dfrac{\sqrt{x}+2}{\sqrt{x}-2}$	$\frac{x+2}{x-2} = \frac{(\sqrt{x})+2}{(\sqrt{x})-2} = \frac{\sqrt{x}+2}{\sqrt{x}-2}$
15)	$d(x+1) = \dfrac{x+3}{x-1}$	$\frac{x+2}{x-2} = \frac{(x+1)+2}{(x+1)-2} = \frac{x+3}{x-1}$

4.3 Composition of Functions and Inverses

The process of taking one function and using it as the input for another function is known as *composition of functions*. The notation uses the symbol " $\circ$ " to show that composition is happening.

Composition Notation

The composition of functions f and g is a function that is denoted by $f \circ g$ and defined as

$$(f \circ g)(x) = f(g(x))$$

When the order is reversed, you get

$$(g \circ f)(x) = g(f(x))$$

This works exactly like the substitution of an expression into a function from the last section (Section 4.2).

Example

Find $(f \circ g)(x)$ when $f(x) = x^2$ and $g(x) = 2x$.

$$(f \circ g)(x)$$
$$\leftrightarrow$$
$$f\big(g(x)\big)$$
$$\leftrightarrow$$
$$f(2x)$$
$$\leftrightarrow$$
$$(2x)^2$$
$$\leftrightarrow$$
$$4x^2$$

Visualization of the Example Above

$$f(x) = x^2 \text{ and } g(x) = 2x$$
$$f\big(g(x)\big) = (2x)^2$$

The function $g(x)$ is inserted into the function $f(x)$.

If you are asked to *evaluate* a composite function, first simplify the composite function and then plug in the number.

Example

Find $(g \circ f)(2)$ when $f(x) = 2x + 3$ and $g(x) = 10x$.

$$(g \circ f)(x)$$
$$\leftrightarrow$$
$$g\big(f(x)\big)$$
$$\leftrightarrow$$
$$g(2x + 3)$$
$$\leftrightarrow$$
$$10(2x + 3)$$
$$\leftrightarrow$$
$$20x + 30$$

Now that $(g \circ f)(x)$ has been found, evaluate $(g \circ f)(2)$

$$(g \circ f)(2)$$
$$\leftrightarrow$$
$$20x + 30$$
$$\leftrightarrow$$
$$20(2) + 30$$
$$\leftrightarrow$$
$$40 + 30$$
$$\leftrightarrow$$
$$70$$

Visualization of the Example Above

$$f(x) = 2x + 3 \text{ and } g(x) = 10x \text{ and } x = 2$$
$$g\big(f(2)\big) = 10(2(2) + 3)$$

In this case, 2 is inserted into the function $f(x)$, which is then inserted into the function $g(x)$.

Inverse Functions

A function that undoes the action of a function f is called the *inverse* of f.

A function g is an inverse of the function f if the domain of g is equal to the range of f:

For every x in the domain of f and every y in the domain of g,

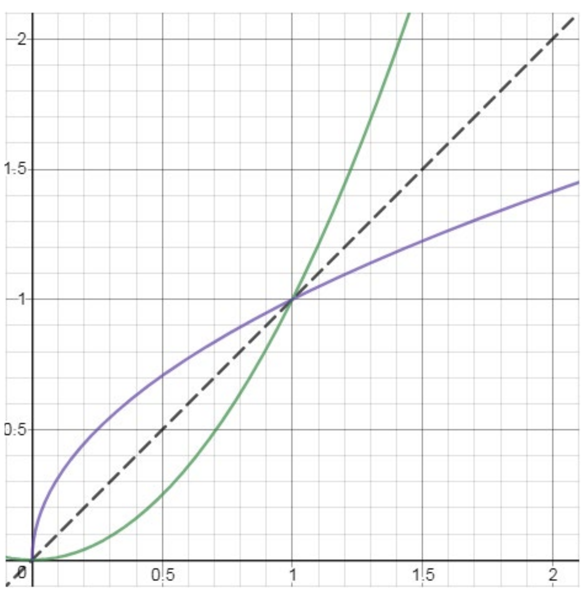
$$g(y) = x \text{ } if \text{ } and \text{ } only \text{ } if \text{ } f(x) = y$$

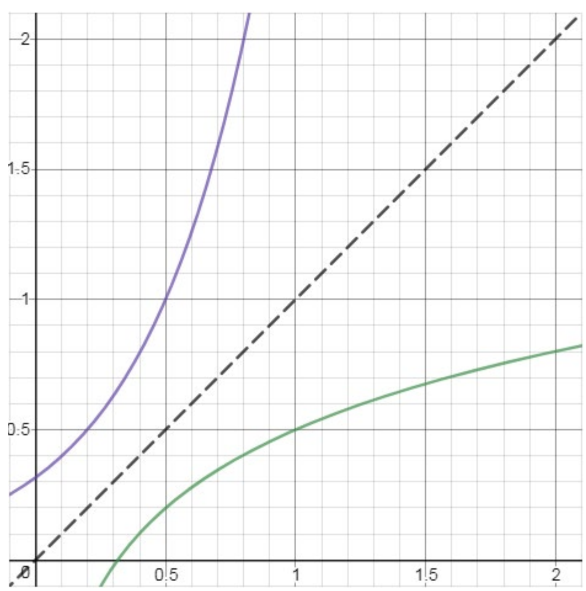
The notation for an inverse function is given by f^{-1}, therefore,

$$f^{-1}(y) = x \text{ } if \text{ } and \text{ } only \text{ } if \text{ } f(x) = y$$

*The notation f^{-1} does not mean $\frac{1}{f}$

Basically, if two functions are inverses of one another, their inputs and outputs are switched. Graphically, this looks like a reflection across the diagonal $y = x$ line.





In the graphs above, the green and purple functions are inverses of one another.

Finding an Inverse Function

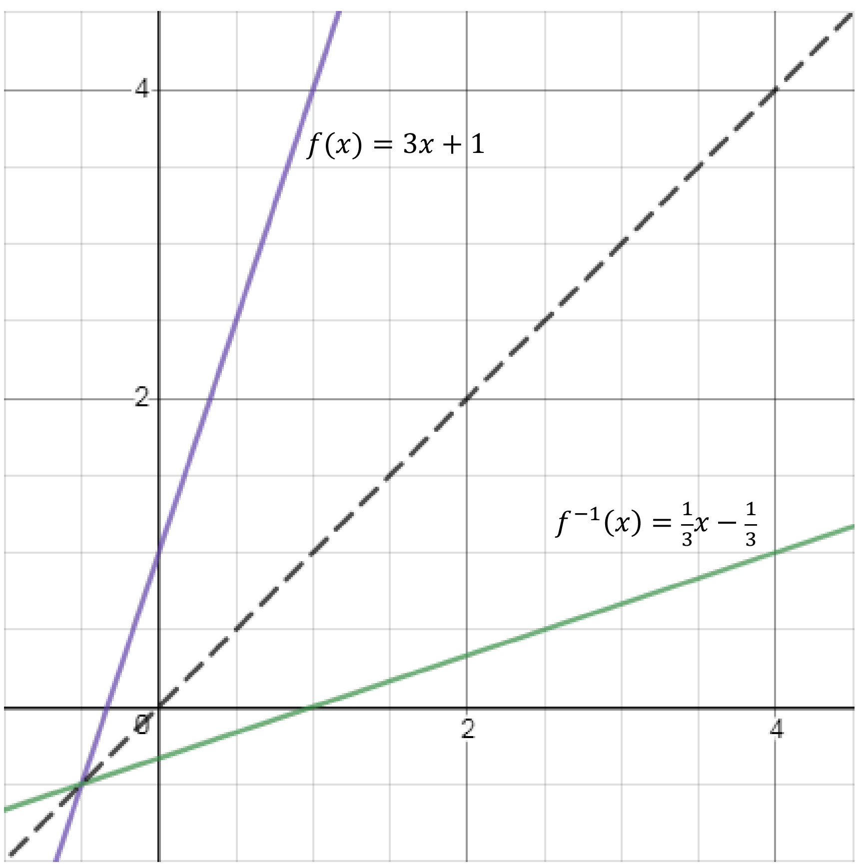
In order to find the inverse of a function, simply switch the x and y values and then solve for y.

Example

Find the inverse function of $f(x) = 3x + 1$.

Original expression	$f(x) = 3x + 1$
Replace $f(x)$ with y	$y = 3x + 1$
Interchange the variables x and y	$x = 3y + 1$
Solve for y	$y = \frac{1}{3}x - \frac{1}{3}$
Final answer, replace y with $f^{-1}(x)$	$f^{-1}(x) = \frac{1}{3}x - \frac{1}{3}$

Graphically, $f(x) = 3x + 1$ and $f^{-1}(x) = \frac{1}{3}x - \frac{1}{3}$ are shown below.



Practice Problems

Evaluate the following.

1) Find $(f \circ g)(x)$ when $f(x) = x^2$ and $g(x) = 5x$.

2) Find $(g \circ f)(x)$ when $f(x) = x^2$ and $g(x) = 5x$.

3) Find $(g \circ g)(x)$ when $f(x) = x^2$ and $g(x) = 5x$.

4) Find $(f \circ g)(x)$ when $f(x) = \frac{1}{x}$ and $g(x) = x + 21$.

5) Find $(f \circ g)(x)$ when $f(x) = x^2 + 2x$ and $g(x) = 2x$.

6) Find $(f \circ g)(2)$ when $f(x) = 2x$ and $g(x) = 4x^2$.

7) Find $(f \circ g)(100)$ when $f(x) = \sqrt{x}$ and $g(x) = x^2$.

8) Find the inverse function of $f(x) = \sqrt{x + 2}$

9) Find the inverse function of $f(x) = 3x + 2$

10) Find the inverse function of $f(x) = \frac{x}{2}$

Answers

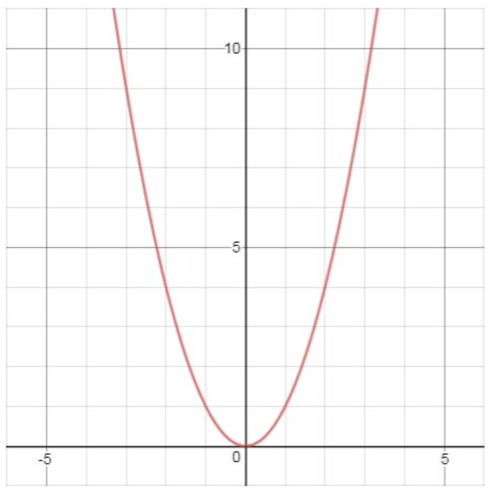
1)	$(f \circ g)(x) = 25x^2$
2)	$(g \circ f)(x) = 5x^2$
3)	$(g \circ g)(x) = 25x$
4)	$(f \circ g)(x) = \frac{1}{x+21}$
5)	$(f \circ g)(x) = 4x^2 + 4x$
6)	$(f \circ g)(2) = 32$
7)	$(f \circ g)(100) = 100$
8)	$f^{-1}(x) = x^2 - 2$
9)	$f^{-1}(x) = \frac{1}{3}(x - 2)$
10)	$f^{-1}(x) = 2x$

4.4 Transformations of Functions

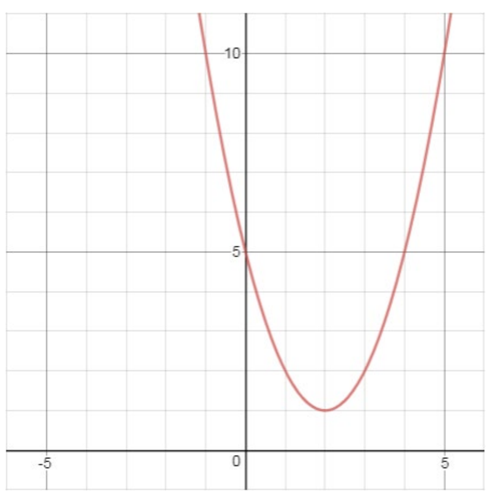
There are ways we can manipulate a function to produce a graph similar to the shape of the original function. We will develop techniques for graphing these functions. These techniques are called transformations.

Example

Let's look at the function $f(x) = x^2$:



We can see that it is centered at the origin $(0,0)$. Now look at the graph of $g(x) = (x-2)^2 + 1$:



The function g is a transformation of the function f.

We can see that the graph of g is identical in shape to that of f but has been shifted 2 units to the right and 1 unit up. This is because the values of -2 and 1 in the function g determined where the vertex was placed. Therefore:

Vertical Shifts

$$g(x) = f(x) + c, if\ c > 0 \quad Raises\ the\ graph\ of\ f\ by\ c\ units$$

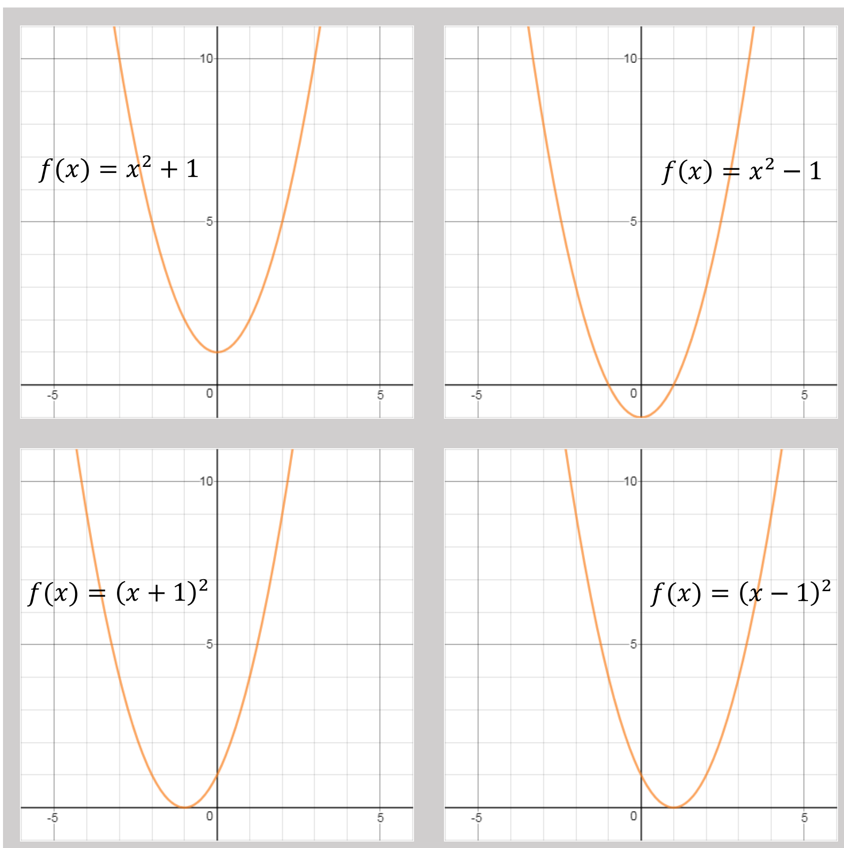
$$g(x) = f(x) - c, if\ c > 0 \quad Lowers\ the\ graph\ of\ f\ by\ c\ units$$

Horizontal Shifts

$$g(x) = f(x + h), if\ h > 0 \quad Shifts\ the\ graph\ of\ f\ to\ the\ left\ h\ units$$

$$g(x) = f(x - h), if\ h > 0 \quad Shifts\ the\ graph\ of\ f\ to\ the\ right\ h\ units$$

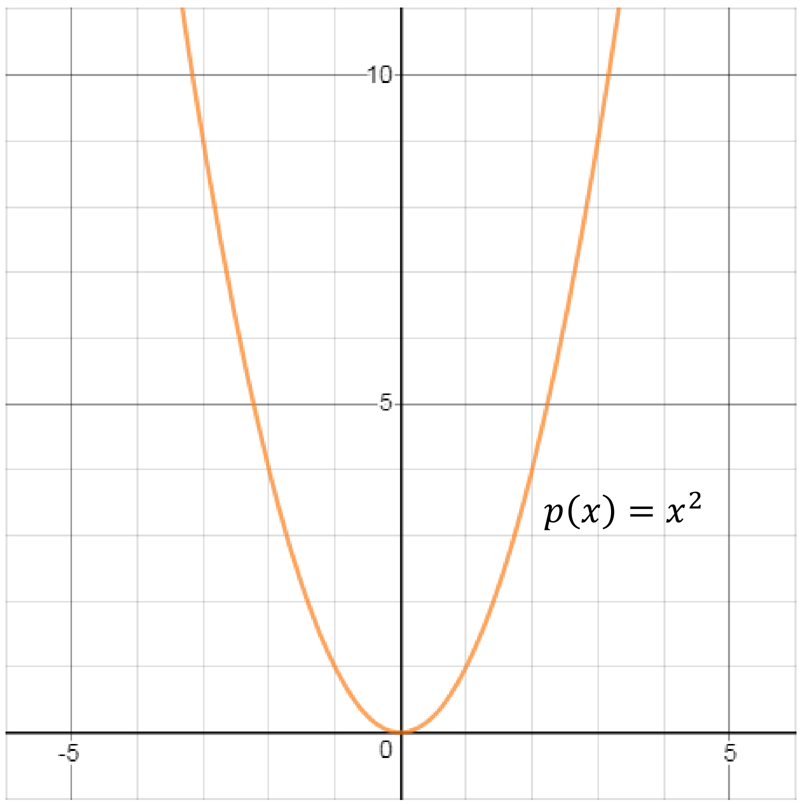
With the function $g(x) = (x - 2)^2 + 1$, the value -2 played the role of shifting the function $f(x) = x^2$ to the right 2 units and the value 1 played the role of raising the function $f(x) = x^2$ up 1 unit. Some more examples are below:



We can also compress and stretch graphs horizontally and vertically. Take a look at the functions:

$$f(x) = 2x^2 \qquad g(x) = (2x)^2 \qquad h(x) = \frac{1}{2}x^2 \qquad k(x) = \left(\frac{1}{2}x\right)^2$$

The functions f and g are only different because of the placement of the 2. Similarly, the functions h and k are only different because of the placement of the $\frac{1}{2}$. To start off, look at the original *parent function*, $p(x)$, without transformations:



On the next page, the different ways this graph can be stretched and compressed will be shown. Compare what you see to this original graph. The differences can be subtle.

Compressing and Stretching Vertically

$g(x) = af(x), if\ a > 0$ — a stretches the graph of f if $a > 1$

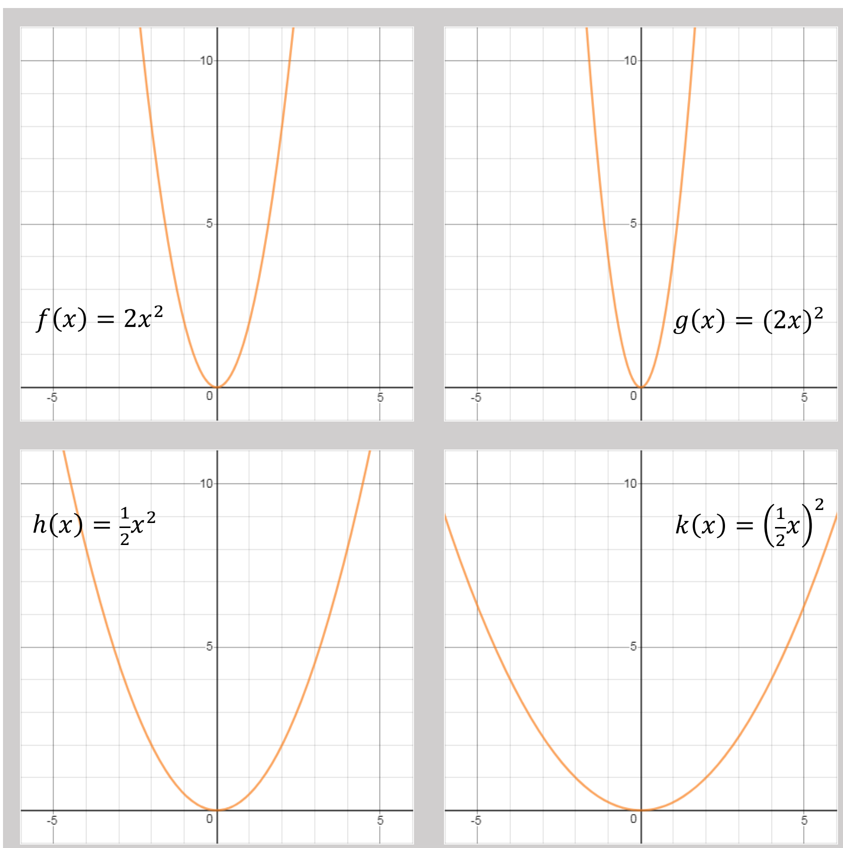
a compresses the graph of f if $0 < a < 1$

Compressing and Stretching Horizontally

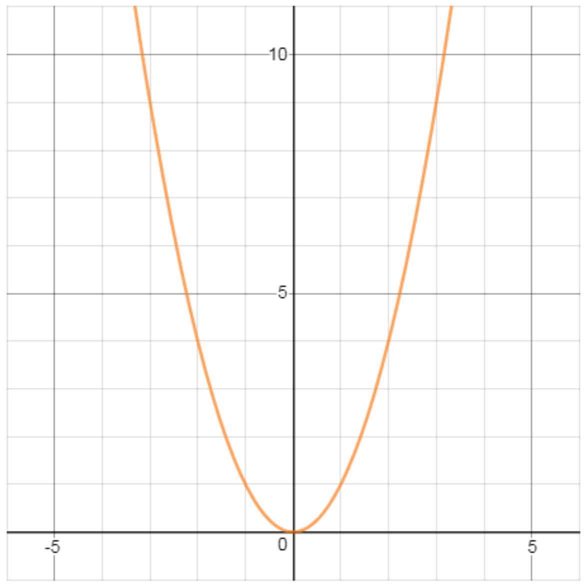
$g(x) = f(ax), if\ a > 0$ — a stretches the graph of f if $0 < a < 1$

a compresses the graph of f if $a > 1$

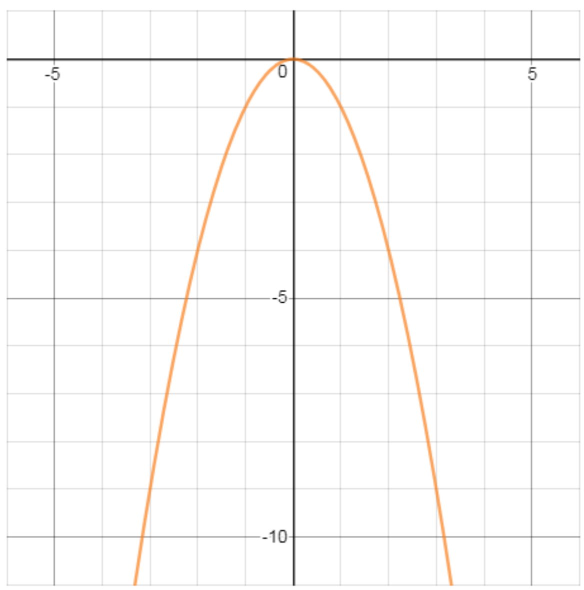
With the function $f(x) = 2x^2$, the value 2 on the outside plays the role of stretching the function x^2 vertically by a factor of 2. With the function $g(x) = (2x)^2$, the value 2 on the inside plays the role of compressing the function x^2 horizontally by a factor of 2. With the function $h(x) = \frac{1}{2}x^2$, the value $\frac{1}{2}$ on the outside plays the role of compressing the function x^2 vertically by a factor of $\frac{1}{2}$. With the function $k(x) = \left(\frac{1}{2}x\right)^2$, the value $\frac{1}{2}$ on the inside plays the role of stretching the function x^2 horizontally by a factor of $\frac{1}{2}$.



Look again at the function $p(x) = x^2$.



Notice the graph has an upward oriented parabola. Now look at the graph $g(x) = -x^2$.



We see that this graph still represents a parabola. However, the shape is oriented downward. The difference between the two functions is the negative sign in front of the original function x^2. This caused a reflection across the x-axis. If the negative had been inside the function like $(-x)^2$, the reflection would happen over the y-axis. In this case with $(-x)^2$ the y-axis reflection looks no different from x^2.

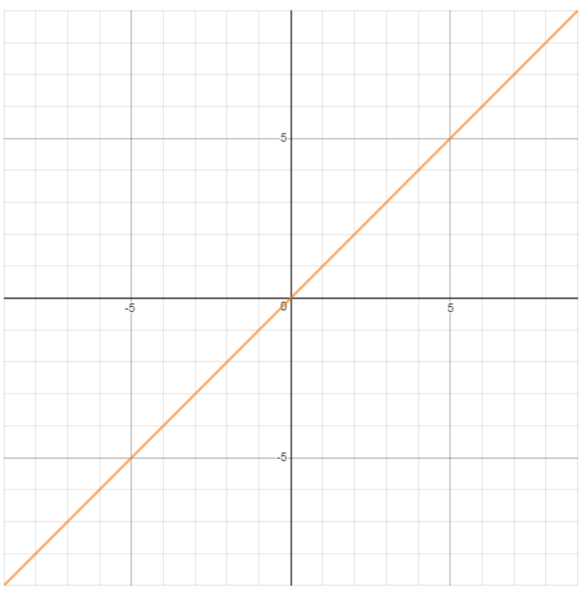
Reflection about the x-axis

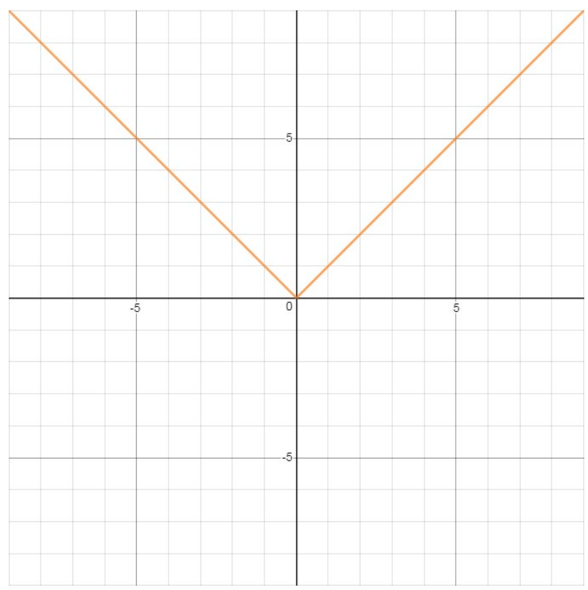
$g(x) = -f(x)$ Multiply $f(x)$ by -1

Reflection about the y-axis

$g(x) = f(-x)$ Replace x with $-x$

You can also transform a function by taking its absolute value as well. Remember the absolute value takes all negative values of an input and produces a positive output. Look at the graph of $f(x) = x$ below (left). For every *negative* value of x we get a *negative* output for $f(x)$. However, notice the difference with the absolute value of $f(x)$ shown below (right). We see that for every *negative* value of x we get a *positive* output of $f(x)$. So basically, taking the absolute value of a function makes all of its output positive.





Practice Problems

Match the function on the left to its corresponding graph on the right.

1) $f(x) = x^2$

2) $f(x) = x^2 - 1$

3) $f(x) = x^2 + 4$

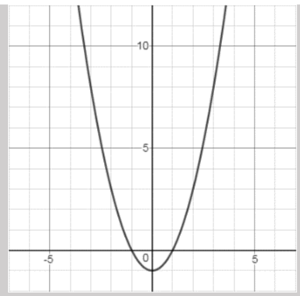
4) $f(x) = (x-2)^2$

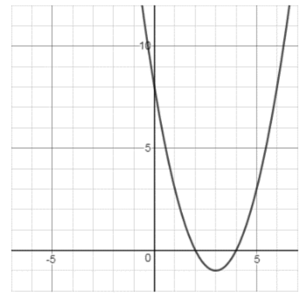
5) $f(x) = (x+2)^2$

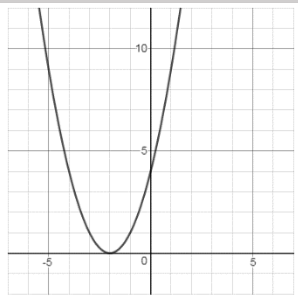
6) $f(x) = 4x^2$

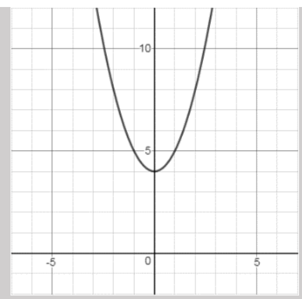
7) $f(x) = \frac{1}{4}x^2$

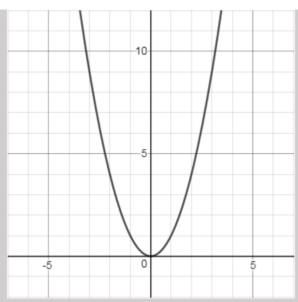
8) $f(x) = (x-3)^2 - 1$

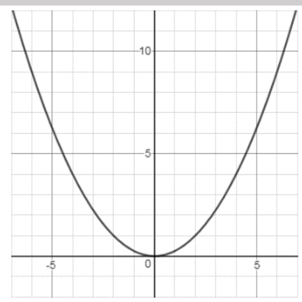
A) 

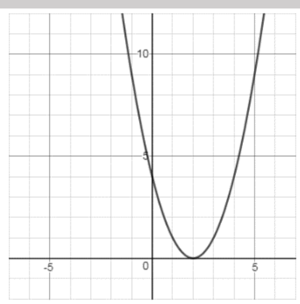
E) 

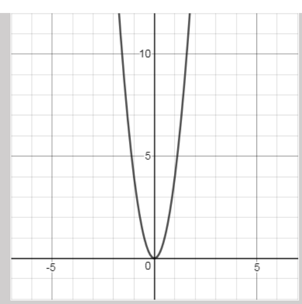
B) 

F) 

C) 

G) 

D) 

H) 

Answers

1)	C
2)	A
3)	F
4)	D
5)	B
6)	H
7)	G
8)	E

5

Quadratic Equations

5.1 Distribution and Foiling

When two quantities are multiplied, all of the pieces being multiplied must be taken into account. Distribution and foiling are the same thing, technically- but they are defined separately so that each process is easier to understand.

Distribution Example

When multiplying two quantities with no pluses or minuses involved, distribution is simple. Multiply like terms together.

$$5 \cdot (2x) = 5 \cdot 2 \cdot x = 10x$$

$$3x \cdot (4x) = 3 \cdot 4 \cdot x \cdot x = 12x^2$$

$$y \cdot (6x) = 6 \cdot x \cdot y = 6xy$$

There are no like terms to combine here, but the multiplication still happens.

Distribution Example

When multiplying two quantities where one has a plus or minus involved, multiply the outer term to each term connected by the plus or minus.

$$5 \cdot (2x + 1) = 5 \cdot 2 \cdot x + 5 \cdot 1 = 10x + 5$$

$$3x \cdot (4x + 2) = 3 \cdot 4 \cdot x \cdot x + 3 \cdot 2 \cdot x = 12x^2 + 6x$$

$$2 \cdot (x - 2) = 2 \cdot x - 2 \cdot 2 = 2x - 4$$

$$-2 \cdot (x - 2) = (-2) \cdot x - (-2) \cdot 2 = -2x - -4 = -2x + 4$$

The negative is carried through the distribution <u>always</u>.

Foiling Example

When multiplying two quantities which both have a plus or minus involved, multiply them in the following way

$(x+1)(x+2) \quad = \quad x^2$

First

$(x+1)(x+2) \quad = \quad 2x$

Outside

$(x+1)(x+2) \quad = \quad 1x$

Inside

$(x+1)(x+2) \quad = \quad 2$

Last

First, outside, inside, last is where the name **Foil**ing comes from.

Foiling Example

This is the result of the above method.

$$(x+1)(x+2) = x^2 + 2x + 1x + 2 = x^2 + 3x + 2$$

Thus the distribution results in $x^2 + 3x + 2$

Practice Problems

Perform the following distributions.

1) $2(3x)$
2) $10(10x)$
3) $x(7x)$
4) $3x(5x^2)$
5) $x(xy)$
6) $6yz^3(2xy^3)$
7) $-x(8x)$
8) $-3(4x^9)$
9) $(x+1)(x+2)$
10) $(x+1)(x+5)$
11) $(x+3)(x+3)$
12) $(x+2)(x-9)$
13) $(x-1)(x+4)$
14) $(x+2)(x-2)$
15) $(x+3)(x-3)$
16) $(x+4)(x-4)$
17) $(x-2)(x-2)$
18) $(x-5)(x-6)$

Answers

1)	$6x$	$2(3x) = 2 \cdot 3 \cdot x = 6 \cdot x = 6x$
2)	$100x$	$10(10x) = 10 \cdot 10 \cdot x = 100 \cdot x = 100x$
3)	$7x^2$	$x(7x) = 7 \cdot x \cdot x = 7 \cdot x^2 = 7x^2$
4)	$15x^3$	$3x(5x^2) = 3 \cdot 5 \cdot x \cdot x^2 = 15 \cdot x^3 = 15x^3$
5)	x^2y	$x(xy) = x \cdot x \cdot y = x^2 \cdot y = x^2y$
6)	$12xy^4z^3$	$6yz^3(2xy^3) = 6 \cdot 2 \cdot x \cdot y \cdot y^3 \cdot z^3 = 12 \cdot x \cdot y^4 \cdot z^3 = 12xy^4z^3$
7)	$-8x^2$	$-x(8x) = -8 \cdot x \cdot x = -8 \cdot x^2 = -8x^2$
8)	$-12x^9$	$-3(4x^9) = -3 \cdot 4 \cdot x^9 = -12 \cdot x^9 = -12x^9$
9)	$x^2 + 3x + 2$	$(x+1)(x+2) = x \cdot x + 2 \cdot x + 1 \cdot x + 1 \cdot 2 = x^2 + 2x + 1x + 2 = x^2 + 3x + 2$
10)	$x^2 + 6x + 5$	$(x+1)(x+5) = x \cdot x + 5 \cdot x + 1 \cdot x + 1 \cdot 5 = x^2 + 5x + 1x + 5 = x^2 + 6x + 5$
11)	$x^2 + 6x + 9$	$(x+3)(x+3) = x \cdot x + 3 \cdot x + 3 \cdot x + 3 \cdot 3 = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$
12)	$x^2 - 7x - 18$	$(x+2)(x-9) = x \cdot x - 9 \cdot x + 2 \cdot x - 2 \cdot 9 = x^2 - 9x + 2x - 18 = x^2 - 7x - 18$
13)	$x^2 + 3x - 4$	$(x-1)(x+4) = x \cdot x + 4 \cdot x - 1 \cdot x - 1 \cdot 4 = x^2 + 4x - 1x - 4 = x^2 + 3x - 4$
14)	$x^2 - 4$	$(x+2)(x-2) = x \cdot x - 2 \cdot x + 2 \cdot x - 2 \cdot 2 = x^2 - 2x + 2x - 4 = x^2 - 4$
15)	$x^2 - 9$	$(x+3)(x-3) = x \cdot x - 3 \cdot x + 3 \cdot x - 3 \cdot 3 = x^2 - 3x + 3x - 9 = x^2 - 9$
16)	$x^2 - 16$	$(x+4)(x-4) = x \cdot x - 4 \cdot x + 4 \cdot x - 4 \cdot 4 = x^2 - 4x + 4x - 16 = x^2 - 16$
17)	$x^2 - 4x - 4$	$(x-2)(x-2) = x \cdot x - 2 \cdot x - 2 \cdot x + 2 \cdot 2 = x^2 - 2x - 2x + 4 = x^2 - 4x - 4$
18)	$x^2 - 11x + 30$	$(x-5)(x-6) = x \cdot x - 6 \cdot x - 5 \cdot x + 5 \cdot 6 = x^2 - 6x - 5x + 30 = x^2 - 11x + 30$

5.2 Factoring Expressions

Factoring is the opposite of foiling. Foiling distributes two quantities into one another, factoring breaks a quantity up into its smaller pieces.

Factoring Example

When factoring an expression, take note what each piece of the expression has in common.

$$5x^4 + 10x^3 + 15x^2$$

$$= 5 \cdot x \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$$

$$= 5 \cdot x \cdot x \cdot x \cdot x + 5 \cdot 2 \cdot x \cdot x \cdot x + 5 \cdot 3 \cdot x \cdot x$$

Identify what each has in common. In this example, each part of the expression has a 5 and an x^2 in common.

Now, rewrite the expression using parentheses. On the left side of the parentheses, we will write the numbers and variables each piece has in common. Inside the parentheses we will write the leftovers of each piece.

$$= 5 \cdot x^2(x \cdot x + 2 \cdot x + 3)$$

$$= 5x^2(x^2 + 2x + 3)$$

The quadratic leftover in the parentheses cannot be simplified further, so we are done.

In conclusion, the factored form of $5x^4 + 10x^3 + 15x^2$ is $5x^2(x^2 + 2x + 3)$.

Another factoring example is given on the next page.

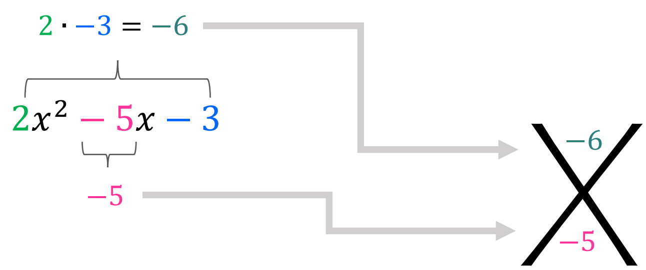
Quadratic Factoring Example

When factoring a quadratic, you can guess and check, or you can use the following process. This process starts by filling out an X diagram that looks like this: 

Consider the quadratic equation $2x^2 - 5x - 3$.

Step 1:

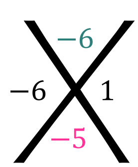
Take the first and last coefficients and multiply them together. Place the result in the top part of the X. Then take the middle coefficient and place it in the bottom part of the X.



Step 2:

Now, find two numbers that multiply together to make the top number, and add together to make the bottom number. In this case, we need two numbers which multiply to make -6 and add to make -5.

The two numbers that meet those conditions are -6 and 1. These numbers are placed in the sides of the X. It doesn't matter which side you put them on.



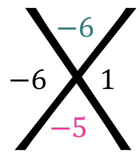
$$-6 \cdot 1 = -6 \quad \textit{and} \quad -6 + 1 = -5$$

(Continued on next page)

Quadratic Factoring Example Continued

Step 3:

Now that the X diagram is filled out, we use that information to rewrite our quadratic equation as follows.



$2x^2 - 5x - 3$ ← Original

$2x^2 - 6x + 1x - 3$ ← New

Notice, the -6 and 1 come from the X diagram.

Step 4:

Our fourth step is to draw a line down the center of the equation and factor each side separately.

$$2x^2 - 6x + 1x - 3$$

Factor this side. →			← Factor this side.
	$2x^2 - 6x$	$1x - 3$	
	$2x(x - 3)$	$1(x - 3)$	

Step 5:

Our final step is to write our factors! One of the factors will be the expression inside the parentheses. Notice how they are both the same! So one of our factors is $(x - 3)$. The other factor is made up of the remaining pieces. In this case, the $2x$ and the positive 1. So our other factor is $(2x + 1)$.

In conclusion, the factored form of $2x^2 - 5x - 3$ is $(x - 3)(2x + 1)$.

Practice Problems

Factor the following expressions.

1) $3x - 9$
2) $5x^2 + 2x$
3) $2x^2 + 4x$
4) $10x^3 + 30x^2$
5) $x^2 + 3x + 2$
6) $x^2 + 5x + 6$
7) $x^2 + 6x + 5$
8) $x^2 + 8x + 16$
9) $x^2 + 6x + 9$
10) $x^2 + 11x + 30$
11) $x^2 - x - 2$
12) $x^2 - 8x + 12$
13) $x^2 - 13x - 30$
14) $5x^2 - 9x - 2$
15) $6x^2 + 13x + 6$
16) $x^2 - 4$
17) $x^2 - 9$
18) $x^2 - 1$
19) $x^2 - 16$

Hint: When you see quadratics in this form, rewrite them with $+0x$ in the middle.

$$x^2 - 4 \quad = \quad x^2 + 0x - 4$$

Answers

1)	$3(x-3)$
2)	$x(5x+2)$
3)	$2x(x+2)$
4)	$10x^2(x+3)$
5)	$(x+1)(x+2)$
6)	$(x+2)(x+3)$
7)	$(x+1)(x+5)$
8)	$(x+4)(x+4)$
9)	$(x+3)(x+3)$
10)	$(x+5)(x+6)$
11)	$(x+1)(x-2)$
12)	$(x-2)(x-6)$
13)	$(x-10)(x-3)$
14)	$(5x+1)(x-2)$
15)	$(3x+2)(2x+3)$
16)	$(x+2)(x-2)$
17)	$(x+3)(x-3)$
18)	$(x+1)(x-1)$
19)	$(x+4)(x-4)$

5.3 Solving Quadratic Equations

Solving quadratic equations requires factoring.

Example

First, use algebra to rearrange the quadratic equation so that it is equal to zero.

$$x^2 = -3x - 2$$

$$x^2 + 3x + 2 = 0$$

Next, factor the left side.

$$x^2 + 3x + 2 = 0$$

$$(x + 2)(x + 1) = 0$$

Now, set each of those expressions equal to zero and solve.

$$x + 2 = 0 \qquad\qquad x + 1 = 0$$

$$x = -2 \qquad\qquad x = -1$$

The solutions to the quadratic equation are $x = -2$ and $x = -1$. Try plugging $x = -2$ into the original equation. Then try plugging $x = -1$ into the original equation. Note you will get $0 = 0$ each time, meaning both values for x satisfy the equation!

Practice Problems

Solve the following equations. (If you have already completed the factoring practice problems in Section 5.2, you can use that work to focus on the solving process here).

1) $x^2 + 5x + 6 = 0$
2) $x^2 + 8x + 16 = 0$
3) $x^2 - x - 2 = 0$
4) $x^2 - 8x = -12$
5) $x^2 = 4$
6) $5x^2 = 9x + 2$
7) $-x^2 = 6x + 9$

Answers

1)	$x = -2, x = -3$	$x^2 + 5x + 6 = 0 \rightarrow (x+2)(x+3) = 0 \rightarrow x + 2 = 0$ *and* $x + 3 = 0$
2)	$x = -4$	$x^2 + 8x + 16 = 0 \rightarrow (x+4)(x+4) = 0 \rightarrow x + 4 = 0$ *and* $x + 4 = 0$
3)	$x = -1, x = 2$	$x^2 - x - 2 = 0 \rightarrow (x+1)(x-2) = 0 \rightarrow x + 1 = 0$ *and* $x - 2 = 0$
4)	$x = 2, x = 6$	$x^2 - 8x + 12 = 0 \rightarrow (x-2)(x-6) = 0 \rightarrow x - 2 = 0$ *and* $x - 6 = 0$
5)	$x = -2, x = 2$	$x^2 - 4 = 0 \rightarrow (x+2)(x-2) = 0 \rightarrow x + 2 = 0$ *and* $x - 2 = 0$
6)	$x = -\frac{1}{5}, x = 2$	$5x^2 - 9x - 2 = 0 \rightarrow (5x+1)(x-2) = 0 \rightarrow 5x + 1 = 0$ *and* $x - 2 = 0$
7)	$x = -3$	$x^2 + 6x + 9 = 0 \rightarrow (x+3)(x+3) = 0 \rightarrow x + 3 = 0$ *and* $x + 3 = 0$

6

Geometry Concepts

6.1 Two-Dimensional Shapes

Geometry word problems consist of relating two things: the information you are given and your knowledge of formulas. Often you will be asked to recall a formula from memory (such as area or perimeter) and use that formula to solve the problem. The following is a list of common 2D shapes and their formulas.

Terminology

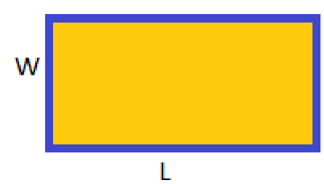
Perimeter: The total distance around the edge of a shape. Finding the perimeter means adding up the lengths of all its sides.	
Area: The size of a surface.	
Hypotenuse: The longest side of a right triangle.	

Rectangles

Perimeter: $P = L + L + W + W$

$= 2L + 2W$

Area: $A = L \cdot W$



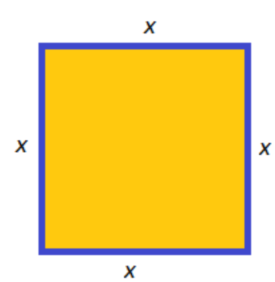
Squares

Perimeter: $P = x + x + x + x$

$= 4x$

Area: $A = x \cdot x$

$= x^2$

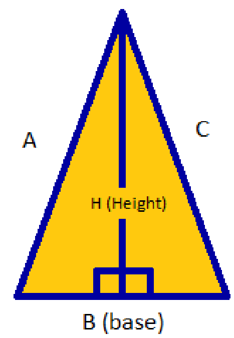


Triangles

Perimeter: $P = A + B + C$

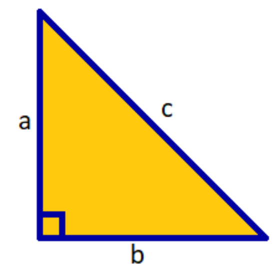
Area: $A = \frac{1}{2} \cdot Base \cdot Height$

$= \frac{1}{2} B \cdot H$



Right Triangles

Pythagorean Theorem: $c^2 = a^2 + b^2$

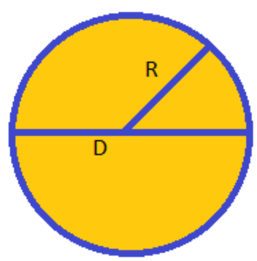


Circles

Circumference: $C = 2\pi R$

Area: $A = \pi R^2$

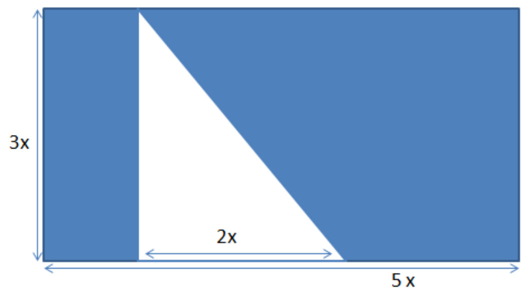
Diameter: $D = 2R$



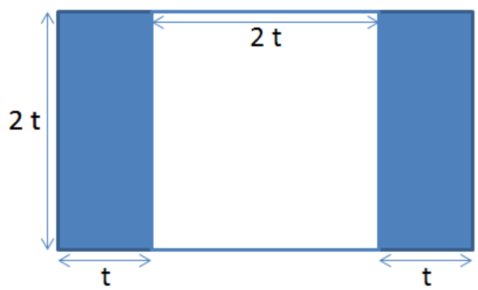
Practice Problems

Solve the following problems.

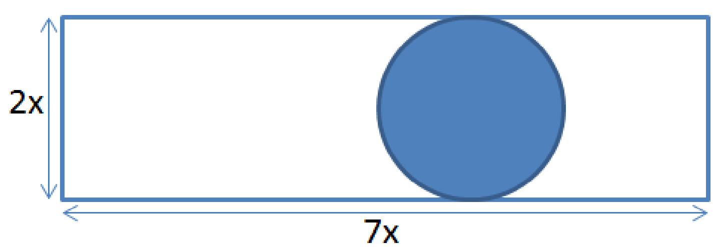
1) Find the area of the shaded region.



2) Find the area of the shaded region.

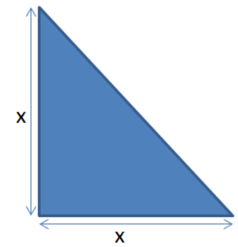


3) Find the area of the shaded region.

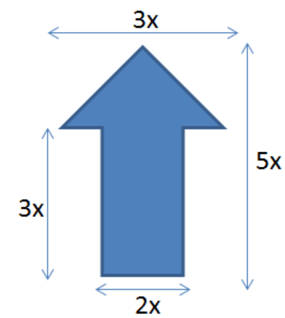


4) If the width of this rectangle is 4 times the length, and the area is 36 units squared, what is the width?

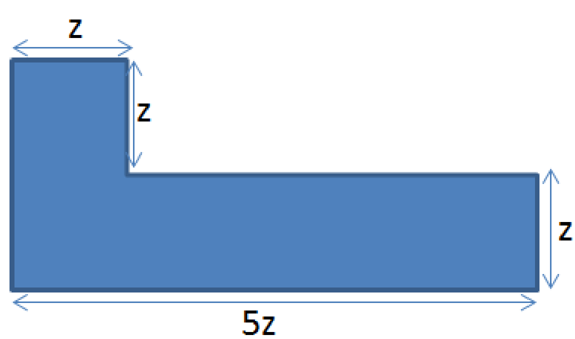
5) If the hypotenuse of the triangle below is $\sqrt{2}$, what is the area of the triangle?



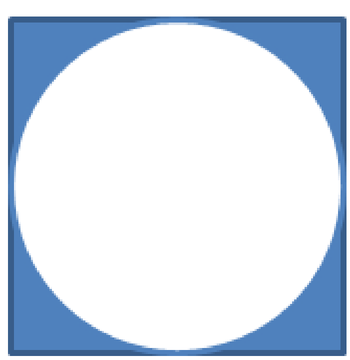
6) The area of the figure below is 144 units squared. What is the value of x?



7) If $z = 2$, find the Area and the Perimeter.



8) Given that the circumference of the circle is $8\,\pi$, what is the area of the shaded region?



Answers

1)	$A = 12x^2$
2)	$A = 4t^2$
3)	$A = \pi x^2$
4)	$w = 12$
5)	$A = \frac{1}{2}$
6)	$x = 4$
7)	$P = 28, A = 24$
8)	$A = 64 - 16\pi$

6.2 Three-Dimensional Shapes

The following is a list of common 3D shapes and their formulas. You are more likely to see cubes and spheres on the test.

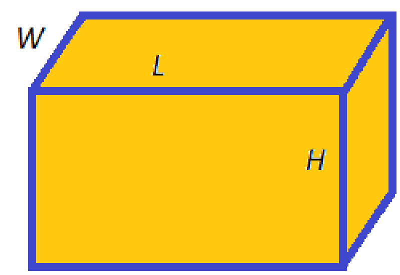
Terminology

Surface Area: The total area of the surfaces of a three-dimensional object.	
Volume: The amount of space inside of a solid figure.	

Rectangular Prisms

Surface Area: $SA = 2(LW + LH + HW)$

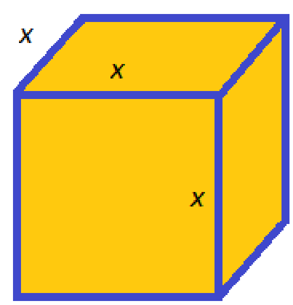
Volume: $V = L \cdot W \cdot H$



Cubes

Surface Area: $SA = 6x^2$

Volume: $V = x^3$



Triangular Prisms

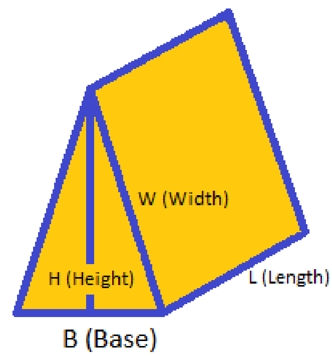
In prisms there are two triangles (front and back), two surfaces (right and left), and one base surface (bottom).

Surface Area: $SA = 2 \cdot \frac{1}{2} BH + 2LW + BL$

$= BH + 2LW + BL$

Volume: $V = Area\ of\ Triangle \cdot length$

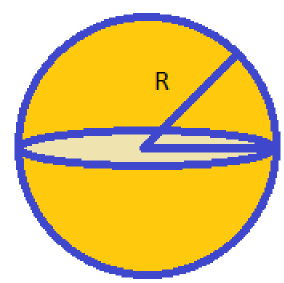
$= \frac{1}{2} B \cdot H \cdot L$



Spheres

Surface Area: $SA = 4\pi R^2$

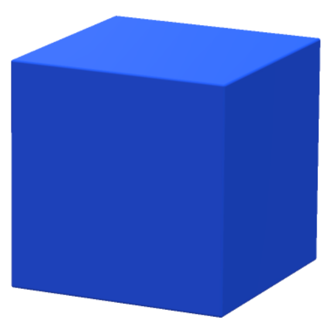
Volume: $V = \frac{4}{3}\pi R^3$



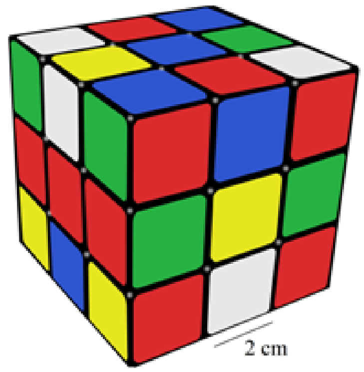
Practice Problems

Find the surface area and volume of the following shapes. Use the information and diagram provided.

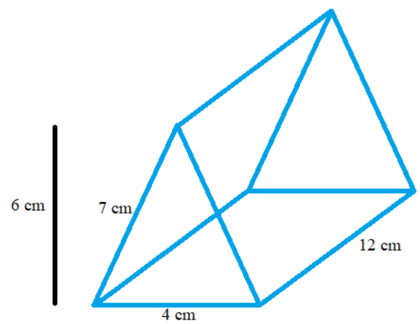
1) The length of the cube is 5 cm.



2) The smaller square of a Rubik's cube has a length of 2 cm.



3) The following is a prism.



4) The soccer ball has a diameter of 9 inches. Find its surface area and volume.

Answers

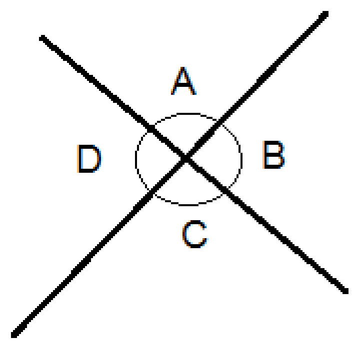
1)	$SA = 150\ cm^2, V = 125\ cm^3$
2)	$SA = 216\ cm^2, V = 216\ cm^3$
3)	$SA = 240\ cm^2, V = 144\ cm^3$
4)	$SA = 81\pi\ in^2, V = 121.5\pi\ in^3$

6.3 Lines and Angles

When lines intersect each other, we can find the angles created by the two lines.

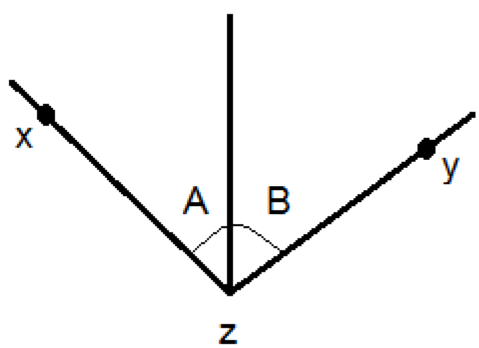
Angles and Parallel Lines

When two lines intersect they form two pairs of opposite angles.



In the image above, A and C are opposite angles and D and B are opposite angles. Opposite angles are always *congruent*, meaning they are equal. If A is 49°, then C is 49°.

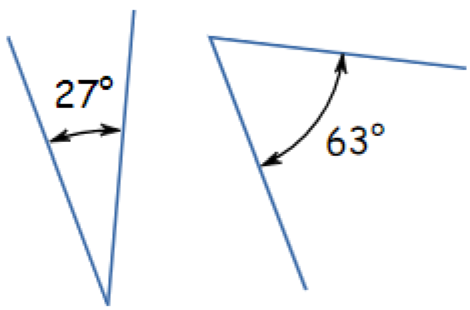
Two angles that are right next to one another are called *adjacent* angles.

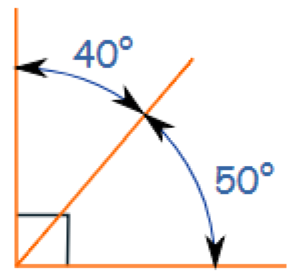


In the image above, the angle xzy is the sum of angles A and B. If A is 30° and B is 60°, then xzy is 90°.

Complementary Angles

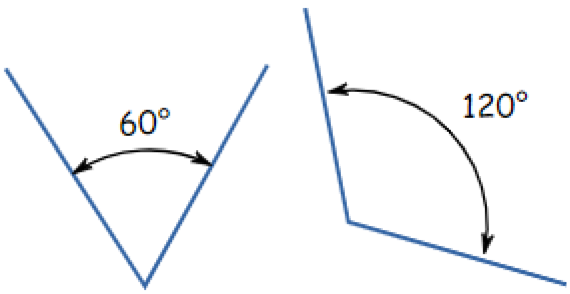
Two angles are Complementary when they add up to 90° (a right angle ⊾).

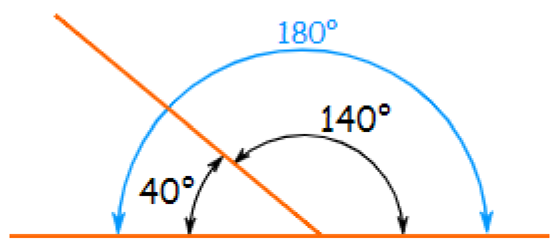




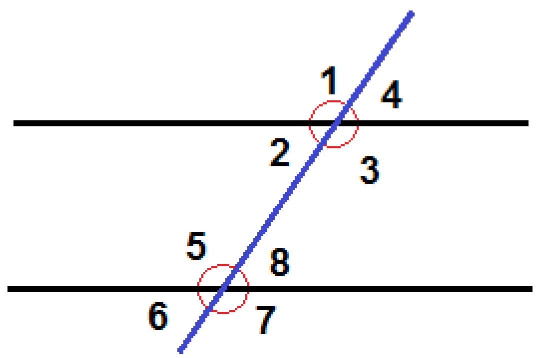
Supplementary Angles

Two Angles are Supplementary when they add up to 180° (a straight angle).

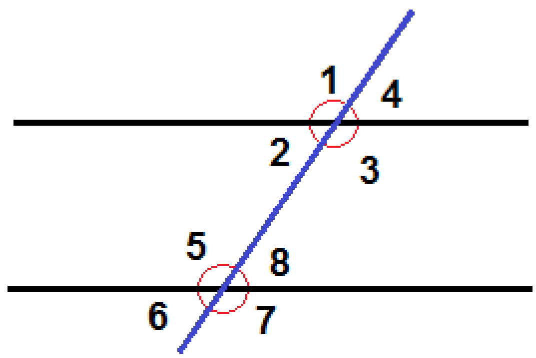




When there are two parallel lines with a third line intersecting them, eight angles are produced. The crossing line (in blue below) is called a *transversal*. An example is shown below.



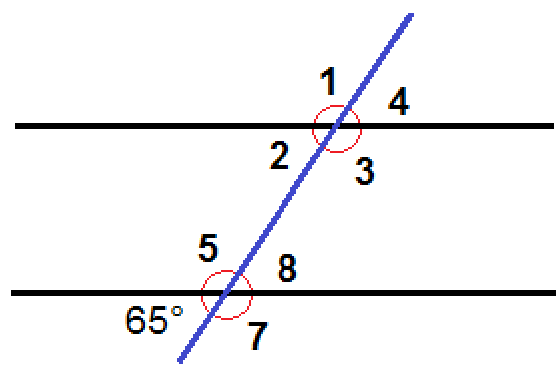
The eight angles created by the transversal form four pairs of corresponding angles. Corresponding angles are congruent.



Angles 1 and 5 are corresponding angles. The pairs 3 and 7, 4 and 8, and 2 and 6 are also corresponding. Angles that are in the area between the parallel lines such as 3 and 5 are called *interior angles*. Angles that are on the outside of the parallel lines like 4 and 7 are called *exterior angles*. Angles on the opposite sides of the transversal like 4 and 5 are called alternate angles.

Example

Which angles in the diagram to the right measure 65°?



Angle 6 is given to be 65°.

Angles 6 and 8 are opposite angles and are therefore congruent. So angle 8 is 65°.

Angles 6 and 2 are corresponding angles and are therefore congruent. So angle 2 is 65°.

Angles 6 and 4 are alternate exterior angles and are therefore congruent. So angle 4 is 65°.

Distance and Midpoint Formula

On the next pages, two formulas will be given. The *distance formula* gives the distance between two points and the *midpoint formula* gives the coordinate point of the center of a line segment.

Distance Formula

Given two points:

(x_1, y_1)

(x_2, y_2)

The distance, d, between those points is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

Find the distance between the points $(2, 5)$ and $(-4, 1)$.

So here $x_1 = 2$, $y_1 = 5$, $x_2 = -4$, and $y_2 = 1$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-4 - 2)^2 + (1 - 5)^2} \\ &= \sqrt{(-6)^2 + (-4)^2} \\ &= \sqrt{36 + 16} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \\ &\cong 7.21 \end{aligned}$$

The distance between $(2, 5)$ and $(-4, 1)$ is $2\sqrt{13}$ units (or about 7.21 units).

Midpoint Formula

Given two points:

(x_1, y_1)

(x_2, y_2)

The midpoint, M, of those points is given by:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

This formula is given in the form of a coordinate point. Once all of the numbers are plugged in and it is simplified, you will be left with a point on a graph.

Example

Find the midpoint between the points $(4, 1)$ and $(10, 5)$.

So here $x_1 = 4$, $y_1 = 1$, $x_2 = 10$, and $y_2 = 5$.

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{4 + 10}{2}, \frac{1 + 5}{2}\right) \\ &= \left(\frac{14}{2}, \frac{6}{2}\right) \\ &= (7, 3) \end{aligned}$$

The point between $(4, 1)$ and $(10, 5)$ is $(7, 3)$.

Practice Problems

Find the distance between the following points.

1) $(0,0)$ and $(1,1)$

2) $(1,1)$ and $(2,9)$

3) $(-1,-3)$and $(-9,-9)$

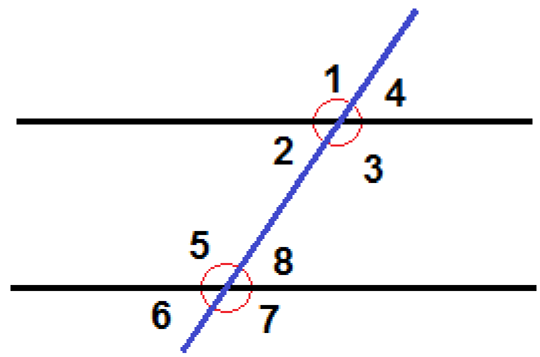
4) $(6,-1)$ and $(-4,4)$

Find the midpoint of the following points.

5) $(1,1)$ and $(2,9)$

6) $(0,3)$ and $(5,2)$

Use the diagram to the right.



7) If angle 8 is 40°, which other angles are 40°?

8) If angle 1 is 100°, which other angles are 100°?

9) If angle 8 is 40°, what is the measure of angle 1?

10) If angle 1 is 100°, what is the measure of angle 6?

Answers

1)	$\sqrt{2}$
2)	$\sqrt{50}$
3)	10
4)	$5\sqrt{5}$
5)	$\left(\frac{3}{2}, 5\right)$
6)	$\left(\frac{5}{2}, \frac{5}{2}\right)$
7)	$Angles\ 2, 4, 6$
8)	$Angles\ 3, 5, 7$
9)	$140°$
10)	$80°$

7

Trigonometry

7.1 Geometric Trigonometry

Trigonometry is the study of triangles. Here are some important equations that help relate different angle units and triangle sides.

Definitions

Both *degrees* and *radians* are ways to measure angles. Degrees are a common unit of measurement you have probably seen before. Radians are numerical values that use π. Radians are often used in higher level math because much of trigonometry involves relating an angle to a circle.

Conversions

Since degrees and radians both measure the same thing, you can convert from one unit of measurement to another.

Degrees → Radians

α is the starting angle in degrees.

$$angle\ in\ radians = \alpha \cdot \frac{\pi}{180}$$

Radians → Degrees

β is the starting angle in radians.

$$angle\ in\ degrees = \beta \cdot \frac{180}{\pi}$$

Example

Convert 30° to radians.

$$30 \cdot \frac{\pi}{180} = \frac{30\pi}{180} = \frac{\pi}{6}$$

The angle 30° in radians is $\frac{\pi}{6}$.

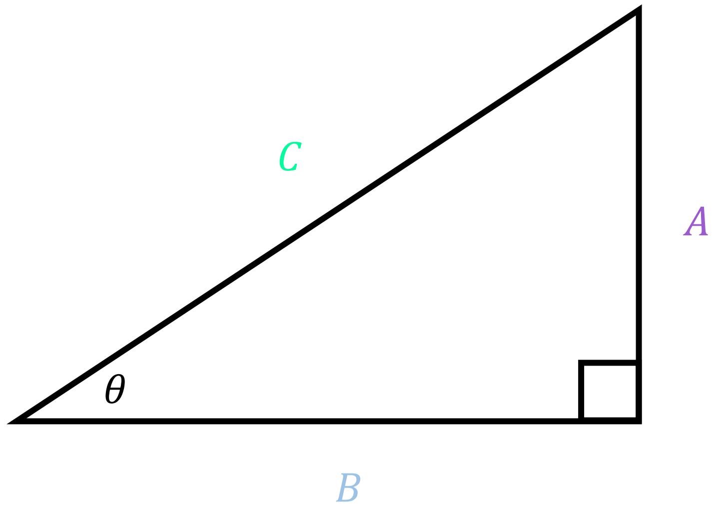
Example

Convert $\frac{2\pi}{3}$ to degrees.

$$\frac{2\pi}{3} \cdot \frac{180}{\pi} = \frac{360\pi}{3\pi} = \frac{360}{3} = 120$$

The angle $\frac{2\pi}{3}$ in degrees is 120°.

Use the following diagram for reference:



Pythagorean Theorem

The *Pythagorean Theorem* relates the three sides of a right triangle in terms of length. The labelling of A and B doesn't matter, but C is always the hypotenuse. This only works for *right triangles*. It is as follows:

$$A^2 + B^2 = C^2$$

Trig Functions

The values of the following trigonometric functions (or just *trig functions*) can be found as follows. The words on the right might help you remember the fractions!

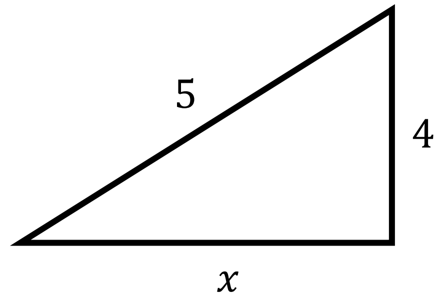
$$sin(\theta) = \frac{opposite}{hypotenuse} = \frac{A}{C}$$ SOH

$$cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{B}{C}$$ CAH

$$tan(\theta) = \frac{opposite}{adjacent} = \frac{A}{B}$$ TOA

Example

What is the length of the missing side on the figure to the right?



Using the Pythagorean theorem, the missing side can be found:

$$A^2 + B^2 = C^2$$

$$4^2 + x^2 = 5^2$$

$$16 + x^2 = 25$$

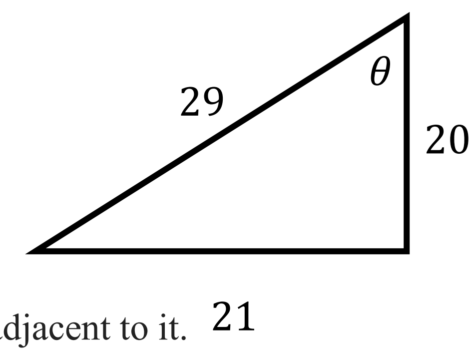
$$x^2 = 25 - 16$$

$$x^2 = 9$$

$$x = 3$$

Example

What is the value of $tan(\theta)$?



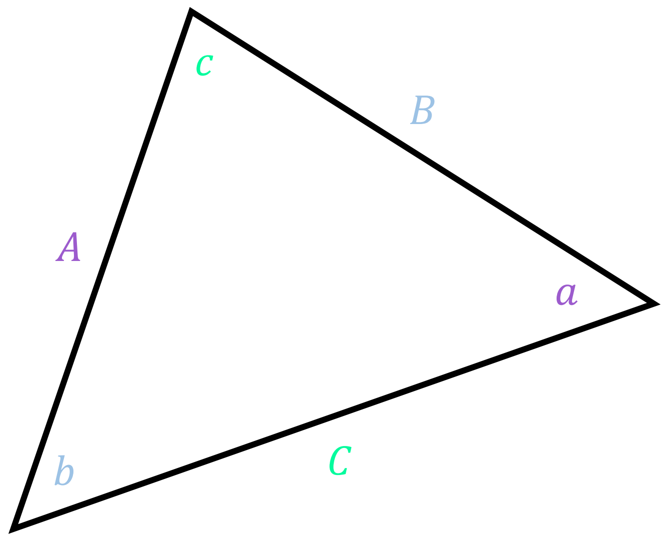
$tan(\theta) = \frac{opposite}{adjacent} = \frac{21}{20}$

Opposite and adjacent are relative to where θ is.
In this case, 21 is directly opposite of θ and 20 is adjacent to it.
The hypotenuse is always the longest side of the triangle, opposite the right angle.
This works the same for all of the trig functions. Each of them is simply equal to the ratio (fraction) of the sides.

For more explanations and examples, ask a tutor for help!

Sometimes you'll have to solve problems for triangles with no right angle.

Use the following diagram for reference:



The Law of Sines and Law of Cosines show relationships between the sides and angles of any triangle (including non-right triangles). Use whichever equations are helpful depending on what information is given!

The Law of Sines

$$\frac{sin(a)}{A} = \frac{sin(b)}{B} = \frac{sin(c)}{C}$$

The Law of Cosines

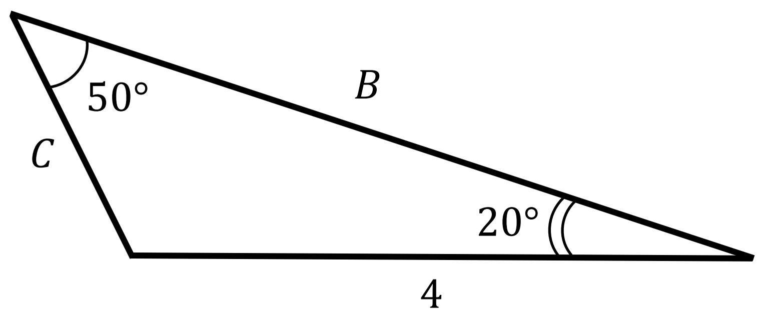
$$A = \sqrt{B^2 + C^2 - (2BC)(cos(a))}$$

$$B = \sqrt{A^2 + C^2 - (2AC)(cos(b))}$$

$$C = \sqrt{A^2 + B^2 - (2AB)(cos(c))}$$

Example

Find the measure of side C.



With the given information, using the Law of Sines is the most appropriate. Side A and its opposite angle a are there, so we can apply the rule to find side C.

$$\frac{sin(a)}{A} = \frac{sin(c)}{C}$$

$$\leftrightarrow$$

$$\frac{sin(50°)}{4} = \frac{sin(20°)}{C}$$

$$\leftrightarrow$$

$$C \cdot \frac{sin(50°)}{4} = sin(20°)$$

$$\leftrightarrow$$

$$C = \frac{4sin(20°)}{sin(50°)}$$

Then, by using a calculator,

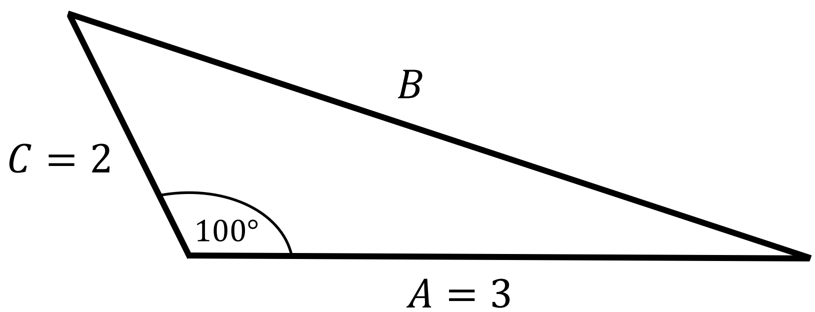
$$C = 1.9$$

So the length of side C is 1.9.

The AAF Accuplacer has a calculator built in for you to use if necessary.

Example

Find the length of side B.



Notice that the Pythagorean Theorem cannot be used here because there is no right angle. With the given information, using the Law of Cosines is the most appropriate. Side A, side C, and angle b are there, so we can apply the rule to find side B.

$$B = \sqrt{A^2 + C^2 - (2AC)(cos(b))}$$

$$\leftrightarrow$$

$$B = \sqrt{3^2 + 2^2 - (2 \cdot 3 \cdot 2)(cos(100°))}$$

$$\leftrightarrow$$

$$B = \sqrt{9 + 4 - (12)(cos(100°))}$$

$$\leftrightarrow$$

$$B = \sqrt{13 - 12cos(100°)}$$

Then, by using a calculator,

$$B = 15.04$$

So the length of side B is 15.04.

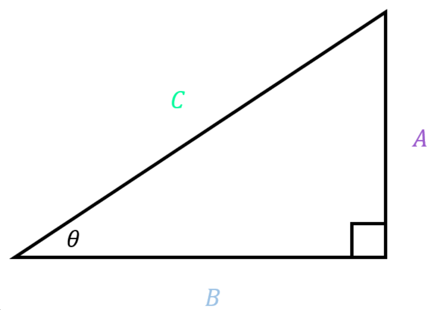
The AAF Accuplacer has a calculator built in for you to use if necessary.

Practice Problems

Convert the following.

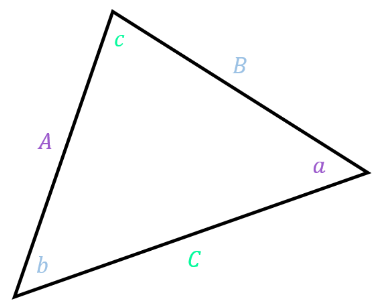
1) Convert 90° to radians.
2) Convert 180° to radians.
3) Convert 225° to radians.
4) Convert $\frac{\pi}{3}$ to degrees.
5) Convert $\frac{5\pi}{6}$ to degrees.
6) Convert 2π to degrees.

Use the triangle to the right for reference.



7) If $B = 3$ and $C = 5$, what is the length of A?
8) If $A = 5$ and $B = 12$, what is the length of C?
9) If $A = 28$, $B = 96$, and $C = 100$, what is $sin(\theta)$?
10) If $A = 28$, $B = 96$, and $C = 100$, what is $cos(\theta)$?
11) If $A = 28$, $B = 96$, and $C = 100$, what is $tan(\theta)$?

Use the triangle to the right for reference. Not to scale.
Round to one decimal place.



12) If $C = 49$, $b = 17°$, and $c = 115°$, what is B?
13) If $b = 29°$, $a = 118°$, and $B = 11$, what is A?
14) If $A = 14$, $B = 10$, and $c = 44°$, what is C?
15) If $A = 52$, $B = 16$, and $c = 115°$, what is C?

Answers

1)	$\frac{\pi}{2}$
2)	π
3)	$\frac{5\pi}{4}$
4)	$60°$
5)	$150°$
6)	$360°$
7)	$A = 4$
8)	$C = 13$
9)	$sin(\theta) = \frac{7}{25}$
10)	$cos(\theta) = \frac{24}{25}$
11)	$tan(\theta) = \frac{7}{24}$
12)	$B = 15.8$
13)	$A = 20.0$
14)	$C = 9.7$
15)	$C = 60.5$

8

Word Problems

8.1 Setting up an equation

An important skill needed on the test is translating sentences into equations.

In this section you will practice translating from a phrase into a mathematical expression. Sometimes this can be straightforward and sometimes it can be very challenging. There is no single perfect strategy to translate, so as you go forward think of these questions:

-Can you explain why the answer you chose is correct?

-If you plugged a number into the expression you chose, would that expression manipulate the number in the way you expect it to?

-Are there any multiple choice answers you can eliminate right away? (Ones that don't make sense or are trying to mislead you).

Addition (+)

When trying to determine if the + operation should be used in an expression, look for key words or phrases like 'sum', 'plus', 'total', 'and', 'together', 'additional', 'added to', 'combined with', 'more than', 'increased by'.

Subtraction (−)

When trying to determine if the − operation should be used in an expression, look for key words or phrases like 'minus', 'reduce', 'difference', 'left', 'change', 'less', 'fewer', 'decreased by', 'less than', 'take away', 'subtract from', 'how much less', 'how much left'.

Multiplication (×)

When trying to determine if the × operation should be used in an expression, look for key words or phrases like 'product', 'times', 'twice', 'doubled', 'tripled', 'per', 'squared', 'cubed', 'of', 'multiplied by'.

Division (÷)

When trying to determine if the ÷ operation should be used in an expression, look for key words or phrases like 'ratio', 'quotient', 'average', 'over', 'into', 'half of', 'a third of', 'split evenly', 'shared equally', 'divided by'.

Practice Problems

1) Which of the following expressions is twice as much as x?

A) $x + 2$
B) x^2
C) $x * x$
D) $2x$

2) Which of the following expressions is 3 times as much as the sum of y and z?

A) $3 * y + z$
B) $3 + y + z$
C) $y + z * 3$
D) $(y + z) * 3$

3) Which of the following expressions is one less than the sum of x and y?

A) $(x + y) - 1$
B) $x * y - 1$
C) $(x - y) + 1$
D) $-1 + (x - y)$

4) Which expression represents 10 more than twice the value of x?

A) $2 + x + 10$
B) $2x + 10$
C) $x + 20$
D) $10x + 2$

5) Which expression represents y minus the sum of x and z?

A) $y - (x + z)$
B) $y + x + z$
C) $y(x - z)$
D) $x - z + y$

6) What is 2 more than the product of x and y?

A) $(x + 2) * y$
B) $2 + x + y$
C) $2 + (x * y)$
D) $(y + 2) * x$

7) Which expression represents the difference between x and y, multiplied by z?

A) $x + y + z$
B) $(x - y) * z$
C) $z * (x + y)$
D) $x - z - y$

8) Which expression represents three times the quotient of x and y?

A) $3 * \left(\frac{y}{x}\right)$
B) $3xy$
C) $3 * x^2y^2$
D) $3 * \left(\frac{x}{y}\right)$

Answers

1)	D
2)	D
3)	A
4)	B
5)	A
6)	C
7)	B
8)	D

8.2 Word Problem Practice

The following is word problem practice. A majority of the test will be interpreting a situation and using your math skills to solve for something.

When you approach word problems, keep these guidelines in mind:

-What information is a distraction (unneeded)?

-What information is given? Can you list it out or should it be written as an equation?

-What does the word problem want you to solve for in the end? (Keep this in mind as you proceed).

-If you feel like you don't have the tools to solve the problem given, is there a way to approximate it?

Practice Problems

Solve the following word problems.

1) Alex the farmer wants to fence off a rectangular area of 100 square feet on their farm and wants the side closest to their house to be 25 feet long. How many feet of fencing do they need to purchase to complete the project?

   A) 29 feet
   B) 58 feet
   C) 100 feet
   D) 75 feet

2) Erin makes $\frac{x}{y}$ dollars selling lemonade, Glenn makes $\frac{x-y}{y}$ dollars as a cook, and James only makes $\frac{2}{3}$ as much as what Glenn makes. Which of the following equations represents the total combined income of all three people?

   A) $\frac{2x-2y}{3y}$
   B) $1+\frac{2x-2y}{3y}$
   C) $1+\frac{6x-3y}{3y}$
   D) $\frac{8x-5y}{3y}$

3) If Lee can walk two miles in one hour, how many miles could they walk in four and a half hours?

A) 8 ¼ miles
B) 8 ½ miles
C) 9 ¼ miles
D) 9 miles

4) If the price of an item is represented by k, how much is the item worth if it is discounted by 23%?

A) $0.23k$
B) $0.77k$
C) $k - 0.23$
D) $k + 0.77k$

5) Micah drops a ball from a cliff that is 200 meters high. If the distance of the ball down the cliff is represented by $D = \frac{1}{2}gt^2$, how long will it take the ball to travel 122.5 meters down the cliff? (Assume $g = 9.8\, {}^{m}/_{s^2}$)

A) 5 seconds
B) 10 seconds
C) 12 seconds
D) 15 seconds

6) If Pat is half the age of Tyler and Tyler is three times the age of Grace, what is Pat's age in 1995 if Grace was born in 1983?

A) 12 years old
B) 16 years old
C) 18 years old
D) 36 years old

7) The area of a right triangle is given by the equation $A = \frac{1}{2}bh$, where b represents the base and h represents the height. If a right triangle has an area of 10 and a height of 5, what is the length of its base?

A) 5
B) 2
C) 4
D) 25

8) In the beginning of the month, Morgan owes Payton, John, and Susan $\$3.50$ each. If Morgan receives a paycheck of $\$250.00$ and repays Payton, John, and Susan, how much will she have left afterwards?

A) $\$239.50$
B) $\$241.50$
C) $\$246.00$
D) $\$243.50$

9) Taylor has $\$90$ in their checking account. They make purchases of $\$22$ and $\$54$. Then they overdraft their account with a purchase of $\$37$ resulting in a $\$35$ fee. The next day, Taylor deposits a check for $\$185$. What is the balance of their bank account after the deposit?

A) $-\$58.00$
B) $\$58.00$
C) $-\$127.00$
D) $\$127.00$

10) The temperatures on a certain planet can be as high as $102°\mathrm{C}$ during the day and as low as $-35°\mathrm{C}$ at night. How many degrees does the temperature drop from day to night?

A) $67°\mathrm{C}$
B) $102°\mathrm{C}$
C) $137°\mathrm{C}$
D) $35°\mathrm{C}$

11) To calculate the Body Mass Index of an individual, their weight (in kilograms) is divided by the square of their height (in meters). If Mike weighs 108 kilograms and stands 2 meters tall, what is his BMI?

A) 27
B) 24
C) 22
D) 18

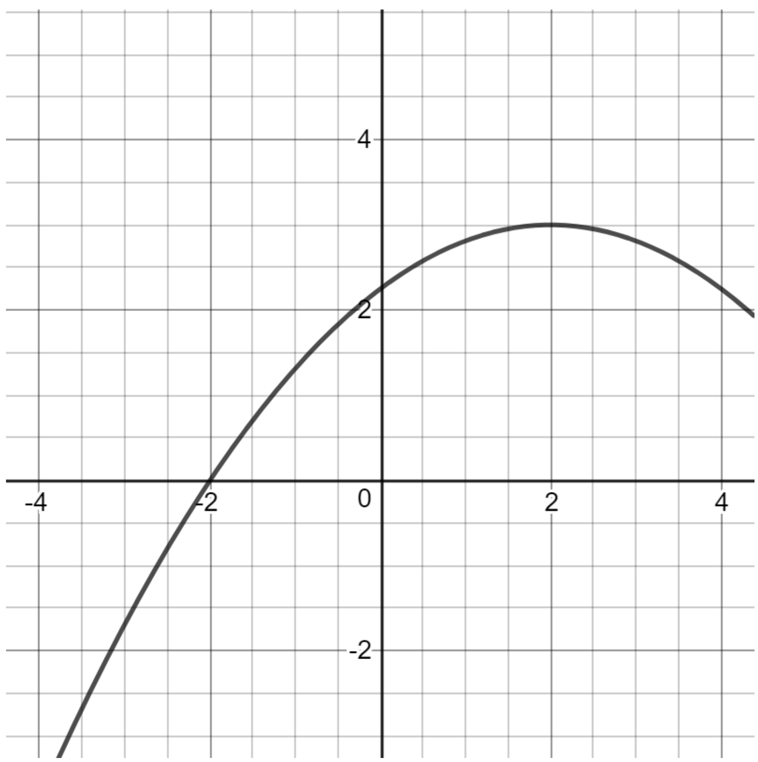
12) Jasmine buys a car at a major dealership. She pays a single payment of $5,775 for it. If 5% of that payment was taxes and fees, what was the actual price of the car? (Round to the nearest dollar)

A) $\$6{,}064$
B) $\$5{,}500$
C) $\$3{,}850$
D) $\$5{,}486$

13) The perimeter of a rectangle is three times the length. The length is six inches longer than the width. What are the dimensions of this rectangle?

A) $12\ in \times 6\ in$
B) $15\ in \times 9\ in$
C) $18\ in \times 10\ in$
D) $10\ in \times 4\ in$

14) What is the area under the curve shown below (above the x-axis)?



A) 10
B) 8
C) 16
D) 12

Answers

1)	B	
2)	D	
3)	D	
4)	B	
5)	A	
6)	C	
7)	C	
8)	A	
9)	D	
10)	C	
11)	A	
12)	D	
13)	A	
14)	C	The trick with this question is to adjust the position of the curve so that the vertex is over $(0,0)$. Once the graph is in that position, you can approximate the area with a triangle. After getting this approximation, choose the answer that is larger than the number you got (since the triangle under-approximates the area).

9

Practice Tests

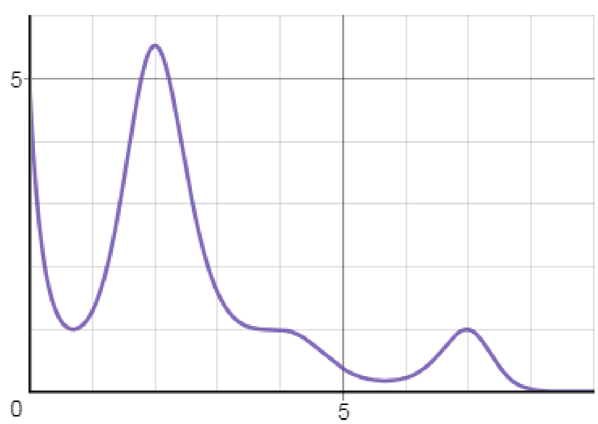
9.1 Practice Test 1

Answer the following questions.

1) What is $45°$ in radians?
   A) $\frac{\pi}{2}$
   B) $\frac{\pi}{4}$
   C) π
   D) $\frac{3\pi}{2}$

2) Find the Domain and Range of $y = \sqrt{x}$
   A) $D: [0, \infty), R: (-\infty, \infty)$
   B) $D: [0, \infty), R: [0, \infty)$
   C) $D: (-\infty, \infty), R: [0, \infty)$
   D) $D: (-\infty, \infty), R: (-\infty, \infty)$

3) The function f is shown on the graph below. For what value of x does f reach its maximum value?



   A) $x = 1$
   B) $x = 5.5$
   C) $x = 2$
   D) $x = 7$

4) A biologist puts an initial population of 500 bacteria into a growth plate. The population is expected to double every 4 hours. Which of the following equations gives the expected number of bacteria, n, after x days? $(24\ hours\ =\ 1\ day)$

A) $n = 500(2)^{x}$
B) $n = 500(2)^{6x}$
C) $n = 500(6)^{x}$
D) $n = 500(6)^{2x}$

5) Solve for x when $\log_{10} 100 = x$.

A) $x = 10$
B) $x = 100$
C) $x = 2$
D) $x = 20$

6) If $x \neq -2$ and $x \neq -\frac{3}{2}$, what is the solution to $\frac{5}{x+2} = \frac{x}{2x-3}$?

A) 3 and 5
B) 2 and $-\frac{3}{2}$
C) -2 and $\frac{3}{2}$
D) -3 and -5

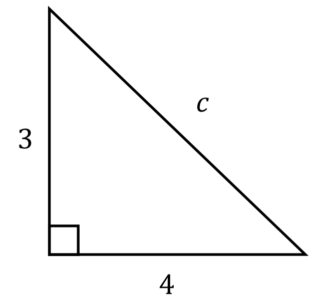
7) Solve the system of equations $\begin{cases} x + y = 7 \\ x + 2y = 11 \end{cases}$

A) $x = 0,\ y = 1$
B) $x = 3,\ y = 2$
C) $x = 3,\ y = 4$
D) $x = 4,\ y = 4$

8) Which of the following expressions is equivalent to $\frac{(3x^4y^{-2})^{-3}}{(2x^3y^2)^{-2}}$?

A) $\frac{27y^{10}}{4x^6}$
B) $\frac{4x^6}{27y^{10}}$
C) $\frac{27x^6}{4y^{10}}$
D) $\frac{4y^{10}}{27x^6}$

9) Use the triangle shown to the right. What is the value of c?



A) $\sqrt{7}$

B) $\sqrt{25}$

C) 7

D) 12

10) What would $f(x) = x^2 + 2x$ look like if it were reflected about the x-axis?

A) $f(x) = -x^2 - 2x$

B) $f(x) = x^2 - 2x$

C) $f(x) = x^2 + 2x$

D) $f(x) = -x^2 + 2x$

Answers

1)	B
2)	B
3)	C
4)	B
5)	C
6)	A
7)	C
8)	D
9)	B
10)	A

9.2 Practice Test 2

Answer the following questions.

1) The perimeter of a rectangle is three times the length. The length is six inches longer than the width. What are the dimensions of this rectangle?
   A) $12\ in \times 6\ in$
   B) $15\ in \times 9\ in$
   C) $18\ in \times 10\ in\pi$
   D) $10\ in \times 4\ in$

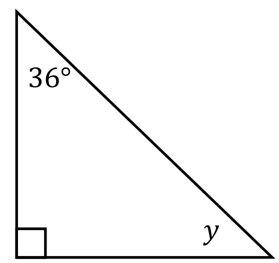
2) Find the distance between the points $(-2, 2)$ and $(2, -2)$.
   A) $\sqrt{2}$
   B) $\sqrt{4}$
   C) $2\sqrt{4}$
   D) $4\sqrt{2}$

3) What is the value of s when $\sqrt{s} = 5$?
   A) $\sqrt{5}$
   B) $\sqrt{25}$
   C) 5^2
   D) 25^2

4) Which of the following is not equivalent to $\frac{x+y}{\sqrt{x}+\sqrt{y}}$?
   A) $\frac{(x+y)(\sqrt{x}-\sqrt{y})}{(x-y)}$
   B) $\frac{(x+y)(\sqrt{x}-\sqrt{y})}{x+y+2\sqrt{xy}}$
   C) $\frac{\sqrt{x}(x+y)-\sqrt{y}(x+y)}{(x-y)}$
   D) $\frac{(x+y)(\sqrt{x}+\sqrt{y})}{x+y+2\sqrt{xy}}$

5) What is $\frac{3\pi}{2}$ in degrees?
   A) $90°$
   B) $180°$
   C) $270°$
   D) $360°$

6) Use the triangle shown to the right. What is the value of y?
   A) $45°$
   B) $54°$
   C) $144°$
   D) $36°$

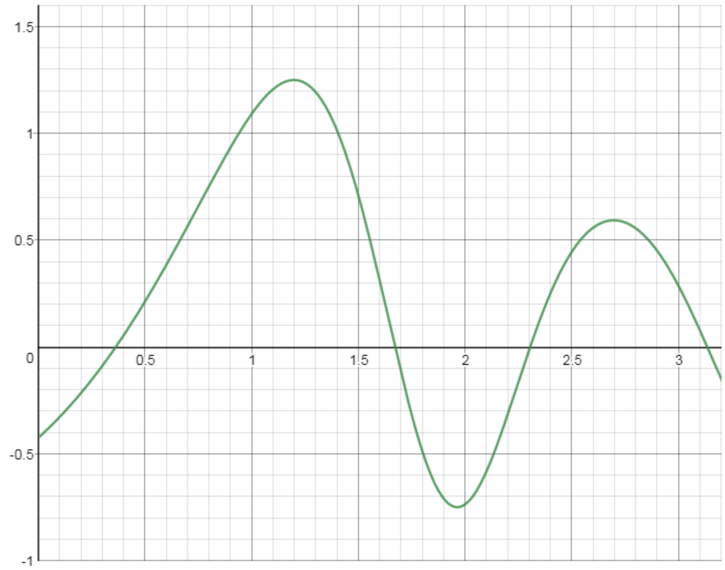


7) Find $k(5)$ for the function $k(x) = 2x + 4$.
   A) 14
   B) 15
   C) 16
   D) 17

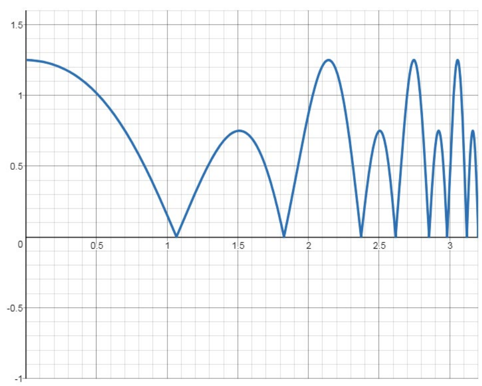
8) In the x-y plane, a line passes though the points (3,5) and the origin (0,0). Which of the following is the equation for the line?
   A) $y = \frac{3}{5}x + 5$
   B) $y = \frac{5}{3}x$
   C) $y = x + \frac{5}{3}$
   D) $y = \frac{5}{3}x + 3$

9) Determine the point at which the equations $6x + 2y = 2$ and $x + 1 = y$ intersect on a graph.
   A) $(0,1)$
   B) $(1,0)$
   C) $(1,-2)$
   D) $(-2,1)$

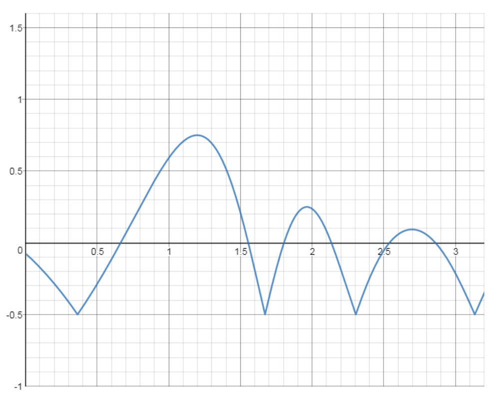
10) The function g is shown on the graph below.



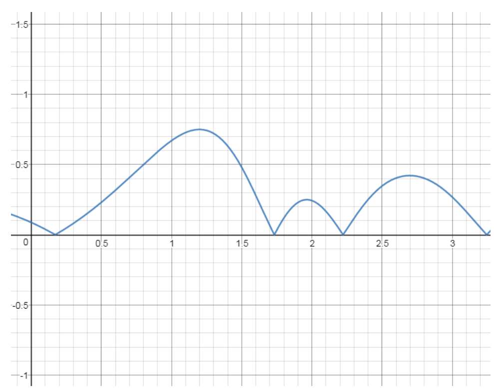
Which of the following is the graph of $|g|$?



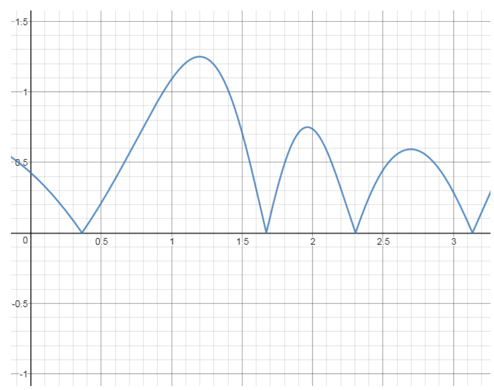
A)



B)



C)



D)

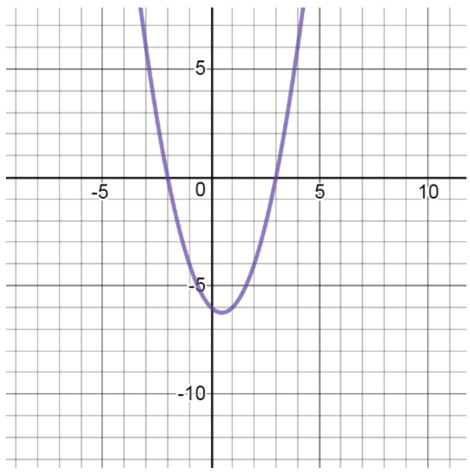
Answers

1)	A
2)	D
3)	C
4)	B
5)	C
6)	B
7)	A
8)	B
9)	A
10)	D

9.3 Practice Test 3

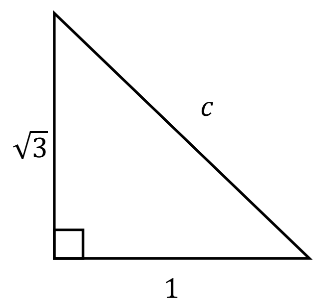
Answer the following questions.

1) What is the equation of the following graph?



A) $y = x^2 - x - 6$
B) $y = x^2 + x - 6$
C) $y = x^2 - x + 6$
D) $y = x^2 + x + 6$

2) Use the triangle to the right. What is the value of c?



A) $\sqrt{4}$
B) $\sqrt{2}$
C) 4
D) 5

3) What is a value of x that satisfies the inequality?

$$18 < 2x^2 < 50$$

A) $x = 4$
B) $x = 5$
C) $x = 20$
D) $x = 25$

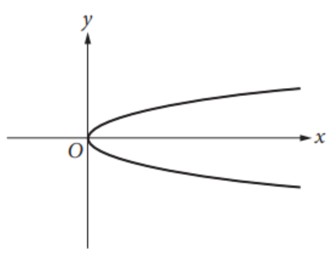
4) Set up the equation for when $\$1000$ is invested at 5% interest compounded annually.

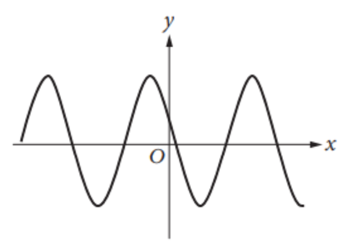
   A) $A(t) = 1000(0.05)^t$
   B) $A(t) = 1000(0.05)^{12t}$
   C) $A(t) = 1000(1.05)^t$
   D) $A(t) = 1000(1.05)^{12t}$

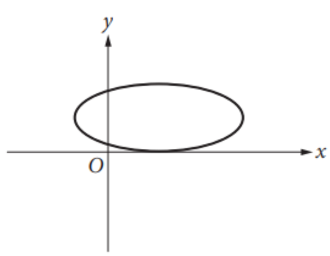
5) The linear equation is in the form $ax + by = c$, where a, b, and c are constants. If the line is graphed in the x-y plane and passes through the origin $(0,0)$, which of the following constants must equal zero?

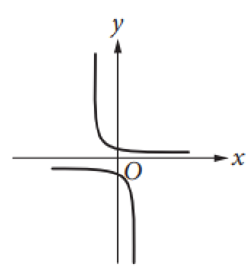
   A) a
   B) b
   C) c
   D) It cannot be determined

6) Which of the following is the graph of a function where $y = f(x)$?

A) 

C) 

B) 

D) 

7) Which of the following best describes the range of $y = -2x^4 + 7$?
   A) $y \leq -2$
   B) $y \geq 7$
   C) $y \leq 7$
   D) $All\ real\ numbers$

8) In the triangle ABC, angle C is a right angle. If $cos(A) = \frac{5}{8}$, what is the value of $cos(B)$?
   A) $\frac{3}{8}$
   B) $\frac{5}{8}$
   C) $\frac{\sqrt{39}}{8}$
   D) $\frac{\sqrt{89}}{8}$

9) For which of the following equations is $x = 6$ the only solution?
   A) $(6x)^2 = 0$
   B) $(x - 6)^2 = 0$
   C) $(x + 6)^2 = 0$
   D) $(x - 6)(x + 6) = 0$

10) In the function $f(x) = a(x + 2)(x - 3)^b$, a and b are both integer constants and b is positive. If the end behavior of the graph of $y = f(x)$ is <u>positive</u> for both very large negative values of x and very large positive values of x, what is true about a and b?
   A) a is negative, and b is even.
   B) a is positive, and b is even.
   C) a is negative, and b is odd.
   D) a is positive, and b is odd.

Answers

1)	A
2)	A
3)	A
4)	C
5)	C
6)	C
7)	C
8)	C
9)	B
10)	D