DMT INSTITUTE

Developing Mathematical Thinking Institute (DMTI)

Professional Development

Curricular Resources

Assessment

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About the DMTI Modules

The DMTI modules are designed to guide classroom instruction and formative assessment for teachers implementing the DMTI curricular materials.

The lessons are not necessarily intended for a single day of instruction. Teachers are encouraged to use their professional judgement regarding pacing. A suggested number of weeks is provided.

DMTI Day Overview

Overall, each module highlights historical and/or cultural themes used to build the lessons. Each Day should start with a warm-up, one or two major components of a lesson, and a take-away.

Components of a DMTI DAY (whether 45, 60, or 90 minutes long)

Warmup (3-5 minutes)

Lesson Component – Problem Solving Situation

Lesson Component – Explanation of math concepts and ideas

Lesson Component – Varied Tasks

Lesson Component – Varied Practice

Takeaway (2-4 minutes)

DMTI Lesson Component Overview

Overall, each module highlights historical and/or cultural themes used to build the lessons. Each Lesson will focus on one or more of the following Lesson Components:

Lesson Component – Problem Solving Situation *(~3 to 10 minutes)*

Lesson Component – Explanation of Math Concepts and Ideas *(~3 to 5 minutes; explanation of math concepts and ideas (with historically, culturally relevant and mathematically accurate ideas)*

Lesson Component – Varied Tasks *(~10-20 minutes; Completed together, in small groups or individually)*

Lesson Component – Varied Practice *(~15-30 minutes; Enactive, Iconic, Symbolic or Context, Iconic, and Symbolic)*

Lesson Review *(After every few lessons a review with different questions – skill, problem solving, conceptual, and justification – will be incorporated as both practice and a formative assessment or checkpoint for teachers.)*

Grade 3

UNDERSTANDING AREA

2-3 WEEKS

Module Sequence

Warm Ups: Mini-lessons to build fluency and understanding

Lesson 1: Which Camp is Best?

Lesson 2: Decomposing Area Models with Equations

Lesson 3: Practice 1

Lesson 4: Extending Understanding of Area

Lesson 5: Examining Errors

Lesson 6: Review

Lesson 7: Practice 2

Warm Up Tasks

BUILDING UNDERSTANDING AND FLUENCY

Warm Up Tasks

The following tasks are intended to be used at the beginning, or in the middle of lessons, in order to increase engagement and to develop students' fluency with mathematical operations.

These tasks are not intended to be taught until mastery. They are meant to be short duration (5-15 minutes) with a high frequency (every lesson) for the purpose of enhancing the mathematical environment of the classroom.

These tasks do not necessarily align with the content of the lessons in this unit. The content addressed by the warm ups varies.

Skip Counting Challenge Game

1. Choose any number to count by (whole numbers or fractions/decimals) and have students count up and back down using that number.

2. Tell students that you will point to yourself and count and that they cannot count out loud when you are pointing to yourself but that when you point to them, they should continue to count on from where you stopped and that you cannot count when you are pointing to them.

3. Practice this process by alternating the counting sequence starting at 0.

4. The goal of the Skip Counting Challenge Game is for the students to follow your counting without making a mistake (e.g. counting out loud when you're not pointing to them) to a given target number. For example, counting by units of $\frac{1}{4}$ from 0 to 5.

Be sure to vary the pattern of where you are pointing so students must attend to where you are counting and think of the counting sequence without speaking out loud. If you are counting by fractions, students can be asked to signal whole number equivalents (e.g. $\frac{12}{4} = 3$) with hand gestures such as "thumbs up" or raised hands.

Counting Using Hand Measurements

1. Place your hands or index fingers together. Ask students to think of a number that could be used to name how far apart your fingers/hands are if there is no space between them. (Answer: 0)

2. Have students count aloud and move their fingers/hands in unison with you as you count by any number (e.g. 4, 12, .5, $\frac{1}{3}$) to a reasonable stopping point.

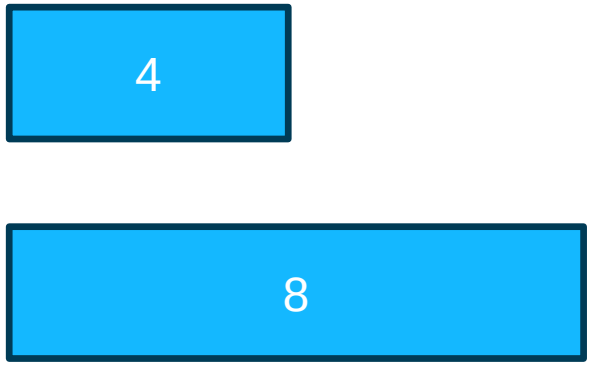
3. Then count backwards from the ending number to 0 following the same process.

4. Try to have students show various reference points or benchmarks in their counting and hand gestures (e.g. 0, 50, 100 if counting by 10's).

Quick Draws

Draw this model.

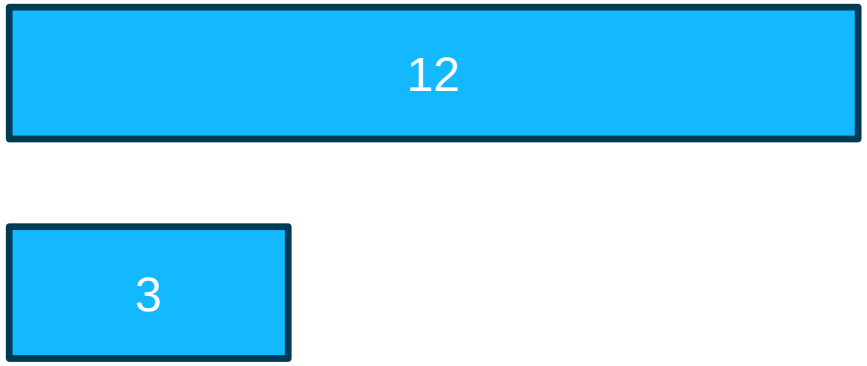
If this is a model of 4, draw a model of what 8 would look like.



Quick Draws

Draw this model.

If this is a model of 12, draw a model of what 3 would look like.

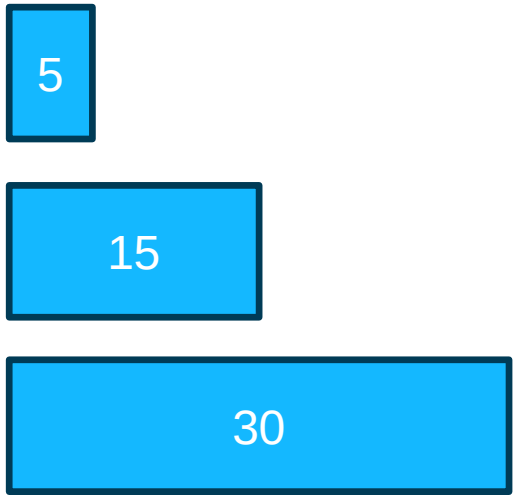


Quick Draws

Draw this model.

If this is a model of 5, draw a model of what 15 would look like.

What would 30 look like?



Quick Draws

Continue the same process with various numbers.

Alternate between giving students a unit that needs to be iterated to compose a larger unit and then a unit that must be partitioned/decomposed to create a smaller unit.

Special emphasis should be given to:

- flexible sizes and shapes of units and especially if using fractions.

- students using the terms *partition*, *iterate*, *compose*, *decompose* and *unit* to describe their drawing strategies.

Shape Drawing

Have students use their index fingers to draw shapes in the air.

Have students turn and move the shapes and ask them to state that the shape is still a (circle, rectangle, square, triangle, etc.) even though it is in different location or different orientation.

Then have students physically “stretch” or “squish” the shape to transform its shape. For example, changing a square into a rectangle (that is not a square) or changing a circle into an oval. Students should describe how they changed the shape and what was the same and different between the two shapes.

Have students use appropriate academic language (e.g. sides, vertices) to describe the shapes.

Challenge students to decompose or partition the drawn shapes and to describe what new shapes are created.

Walking Paths

1. Have 3-4 students stand next to you in a line with all of your backs up against a wall or specific landmark in the room.

2. Tell the class that they will be counting how many steps away from the landmark you and the 3-4 students are.

3. Ask the class, *“If we haven’t taken any steps, what number should we say that means we haven’t gone anywhere yet? What number means nothing?”* (Answer: 0).

4. Students should be guided to count the steps away from the landmark you and the 3-4 students take only as your feet touch the ground. Guide the 3-4 students with you by saying, *“Step forward,”* quietly for each step. As a class, agree on the unit that each step will represent. These units could be whole numbers, decimals or fractions.

Walking Paths

4. Students should be guided to count the steps away from the landmark you and the 3-4 students take only as your feet touch the ground. Guide the 3-4 students with you by saying, *“Step forward,”* quietly for each step. As a class, agree on the unit that each step will represent. These units could be whole numbers, decimals or fractions.

5. After several steps have been taken, have the students walking with you step backward. Guide the class to recognize that they are now counting backwards as you are measuring the distance you are from the landmark. Continue this process until you arrive at the starting location, which would be 0.

6. Repeat this process in small groups and then have students do this in small groups without direction once they are familiar with the directions for the activity.

Lesson 1

WHICH CAMP IS BEST?

Lesson 1: Which Camp is Best?

The Nez Perce tribe lived near Lapwai, Idaho. The picture to the right is a Nez Perce camp in the year 1899. There were 200 people living in the camp.

The Nez Perce lived off a combination of farming, fishing and hunting. It was very important to choose campsites to settle in that would have plenty of food for the tribe.

Lesson 1: Which Camp is Best?

To the right is a map of the territory the Nez Perce lived in over the many years they traveled in the northwest part of the North American continent.

The green section is the land they spent the most time in and currently live in.

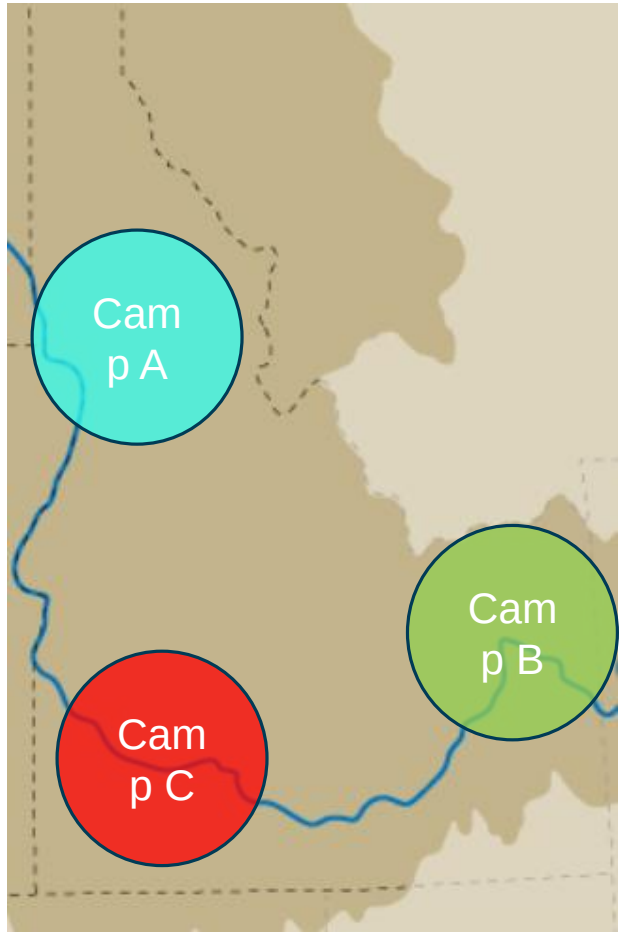
Lesson 1: Which Camp is Best?

Here is a closer look at the territory the Nez Perce were living in during the late 1800's and early 1900's.

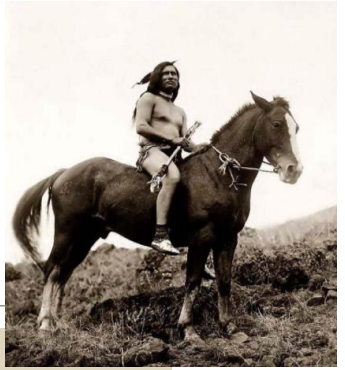
There are 3 camp sites shown. You are going to work on a problem that the Nez Perce had to solve over many years.

Journal: If the tribe was considering moving to one of the camps, which camp would be best and why?

The sites are listed as Camp A, Camp B and Camp C.



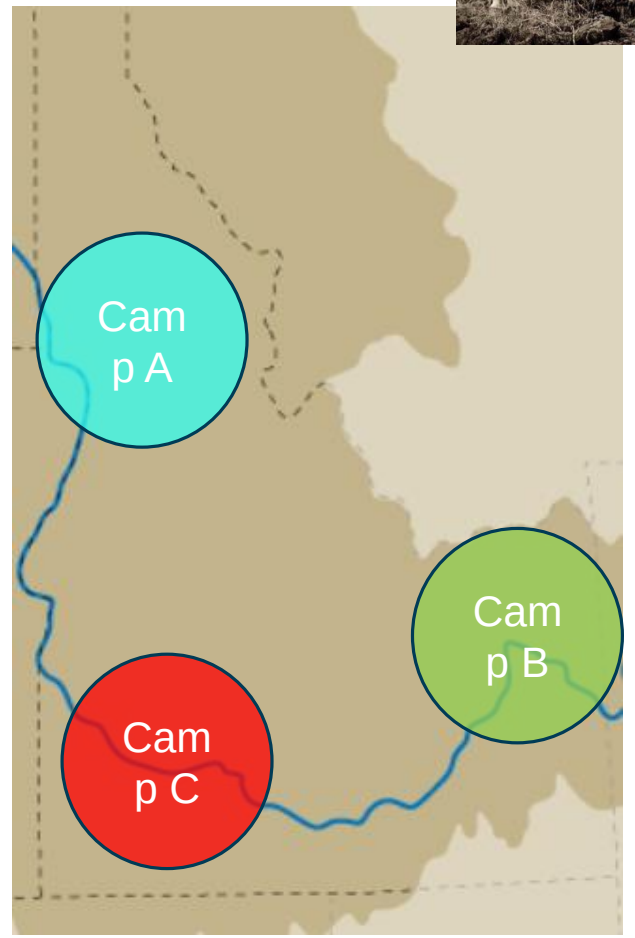
Lesson 1: Which Camp is Best?



To help decide which camp would be the best choice for the tribe, the Nez Perce sent scouts to collect information about each of the camp sites.

The first thing the scouts were asked to look at was the land available and the space the hills, mountains, lakes, rivers and streams left for living space.

We call all of these parts of the land the ***terrain***. We are also measuring the attribute of ***area*** when we look at how much space is covered by shapes or boundaries.

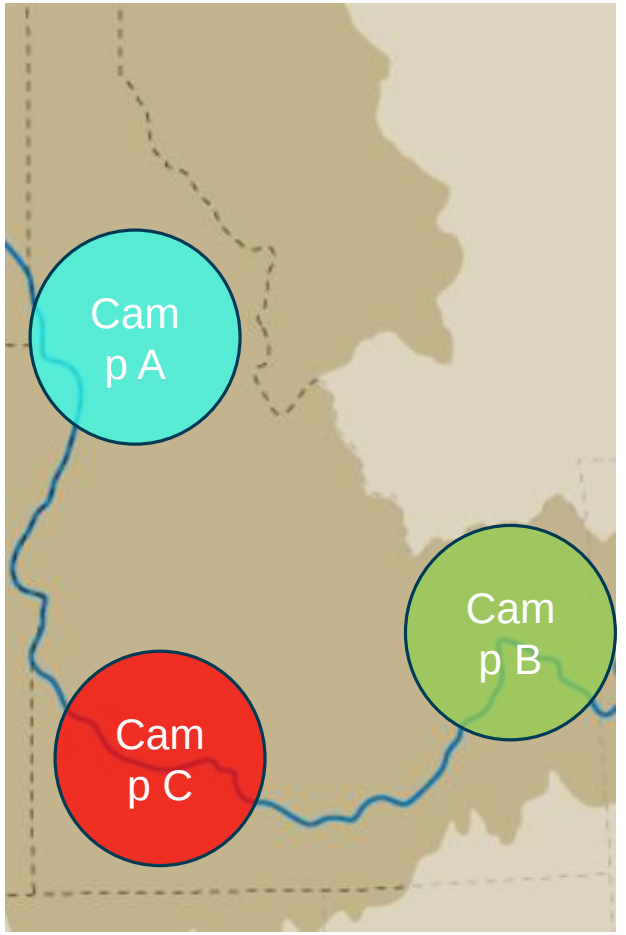


Lesson 1: Which Camp is Best?

Now we will look at the area of the campsites to decide which camp makes the most sense for the Nez Perce to choose.

The Nez Perce did not use square units to measure area, but they did examine the open space available for camps by using landmarks such as trees and hills and thinking about the space between these landmarks.

This is a way to think about area as "covering flat space."



Lesson 1: Which Camp is Best?

Even though the Nez Perce did not use square units to measure area, we are going to explore the area of the campsites using square units.

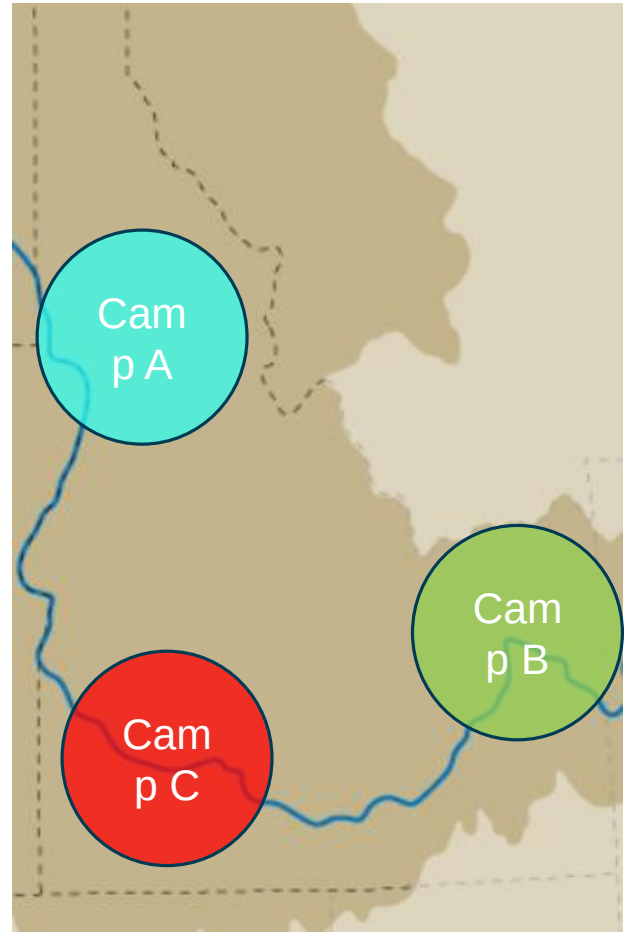
Camp A measures 40 square units.

Camp B measures 41 square units.

Camp C measures 36 square units.

Using the graph paper, draw figures composed of rectangles that have the appropriate areas for each campsite.

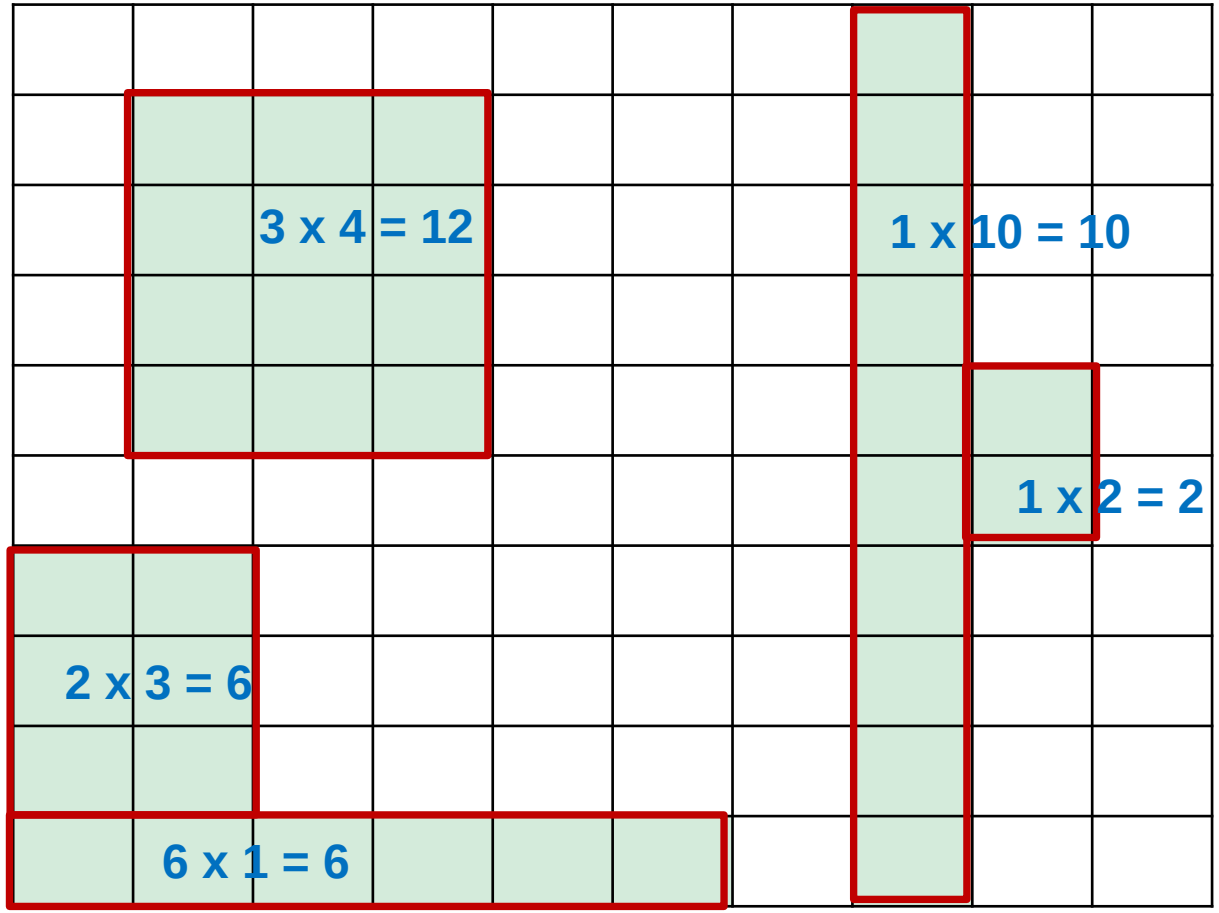
Create maps of the area that would lead the Nez Perce to chose each of the camps if their goal was to the have most open space that was closest to a large square.



Examples of Area Models

*In two of these models, the area has been decomposed into rectangles. **Decompose** is a word we use to tell us we are breaking something apart.*

Here are some examples of how to draw and label area models that can be helpful for the task.
These examples are for only 12 square units and do not use 40, 41 or 36 square units like the problem we are solving does.
Think about the way flat space can be arranged in the real world. Notice that not all examples are a single rectangle.



Journal: What patterns do you notice about the side measurements of the rectangles and the matching multiplication equation?

***Answer:** We can multiply the side measurements to find areas of rectangles.*

Lesson 1: Summary

1. Why do you think you can multiply two of the side measurements of a rectangle to find the area?

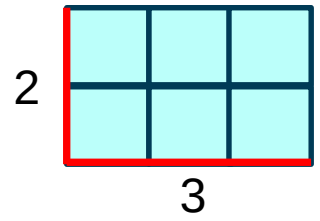
2. What would the area be of a rectangle that was 8 units wide and 4 units high?

3. What is something in the room you could measure the area of?

Lesson 1: Summary

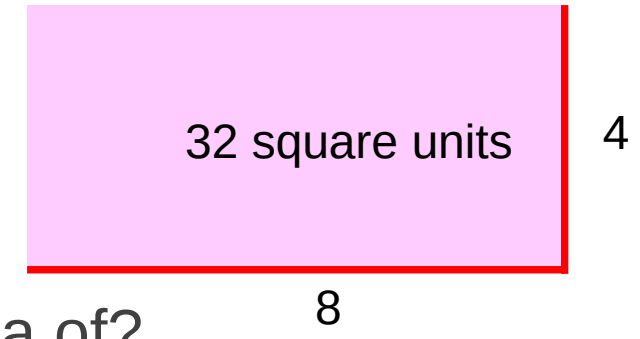
1. Why do you think you can multiply two of the side measurements of a rectangle to find the area?

We can multiply the side measurements to find areas of rectangles because the sides are lengths of the square units composing the area. We can iterate, or copy) a row of 3 square units 2 times and that will create a 2 x 3 rectangle with an area of 6 square units.



2. What would the area be of a rectangle that was 8 units wide and 4 units high?

4 x 8 = 32 and 8 x 4 = 32, so a rectangle with a width of 8 and a height of 4 would measure 32 square units of area.



3. What is something in the room you could measure the area of?

Lesson 2

DECOMPOSING AREA MODELS WITH EQUATIONS

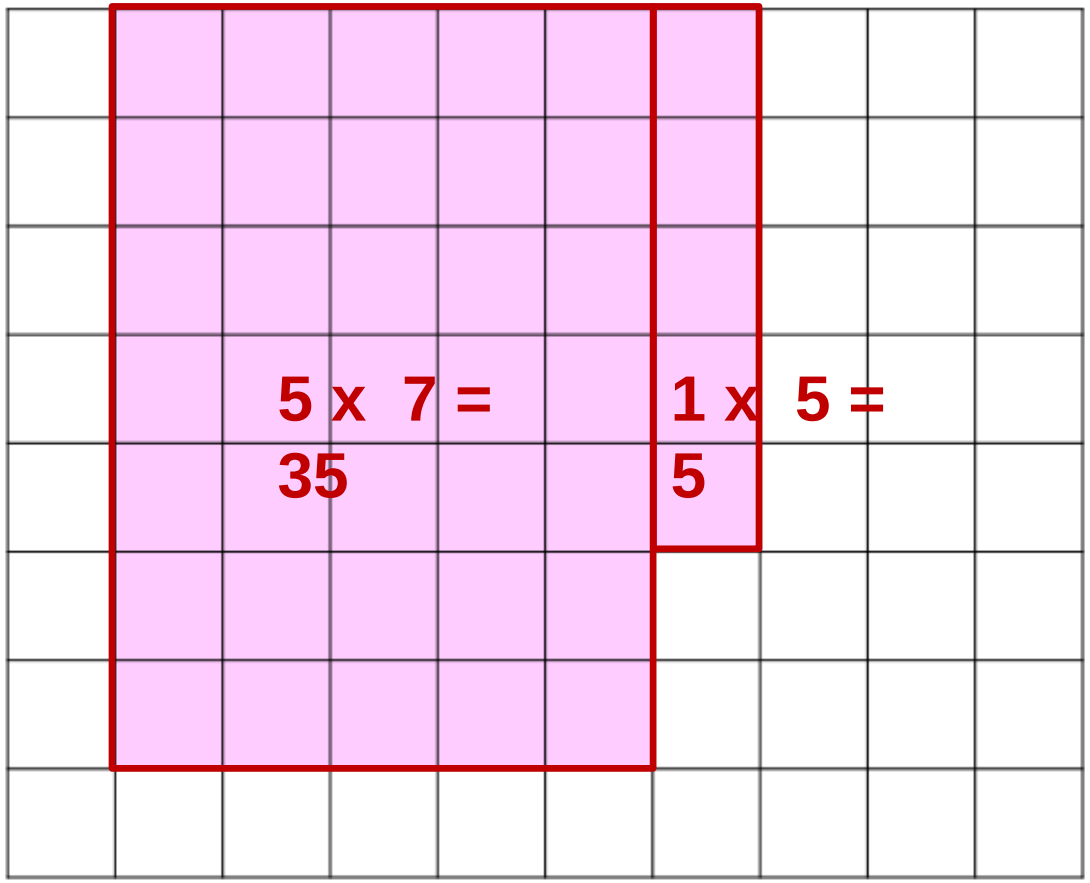
Lesson 2: Area Models and Equations

With a partner, describe how to ***decompose*** this area into rectangles.

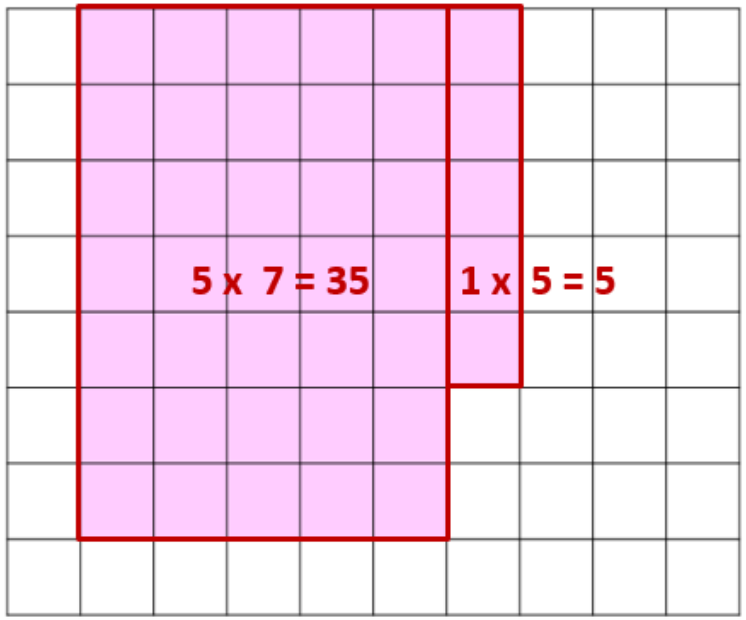
There are several ways to decompose the area into rectangles in order to find the area of 40 square units.

Here is one example you may have discussed.

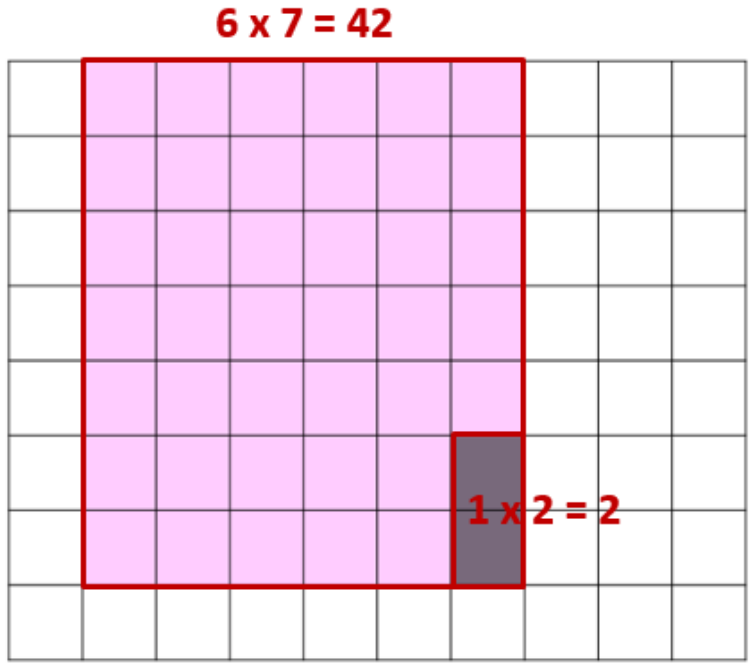
Task: How could you use a 6 x 7 area model to still find the area of this 40 square unit area?



Lesson 2: Area Models and Equations



In this model, we decomposed the area into 5 x 7 and 1 x 5 areas. We can notate this mathematically this way:
(5 x 7) + (1 x 5) = 35 + 5 = 40



But in this model, we started with a larger area and then subtracted the extra area we don't need. We can notate this mathematically this way:
(6 x 7) - (1 x 2) = 42 - 2 = 40

Lesson 2: Area Models and Equations

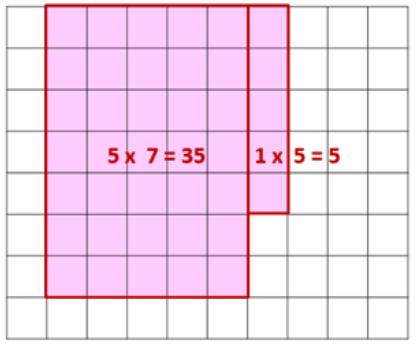
Now that we know how to use equations to describe area models, let's try a new challenge.

1. Try to come up with as many different ways to compose an area of 27 square units.

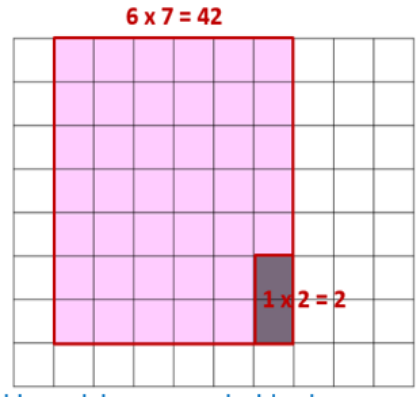
2. Use the graph paper to show your models.

3. Write equations to match the different ways you are finding the area.

Challenge yourself to use both addition and subtraction in your equations, just as we did earlier.



(5x7) + (1x5) = 35 + 5 = 40



(6x7) - (1x2) = 42 - 2 = 40

Lesson 2 Review: Exit Ticket

Answer all three questions. Then, choose one to write your response down on a piece of paper you will turn in.

4. How can you find the area of shapes by decomposing them?

5. If you find the area of a larger figure, what do you do to find the area of shapes within that larger figure?

6. Explain why we use square units to measure area.

Lesson 3

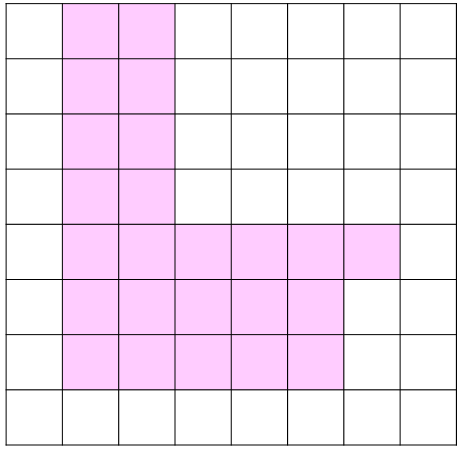
PRACTICE 1

Lesson 3: Practice

Complete the Lesson 3 worksheet by doing the following:

- Fill in the missing parts of the practice grid.
- If you are given a model, write ***at least two*** equations.
- If you are given equations, draw a model that matches the equations used to find the area.

Name:_____

Models	Equations	Area
1. 		
2.	(6 x 5) + (4 x 5) = 30 + 20 = 50 (6 x 10) – (2 x 5) = 60 – 10 = 50	

Lesson 4

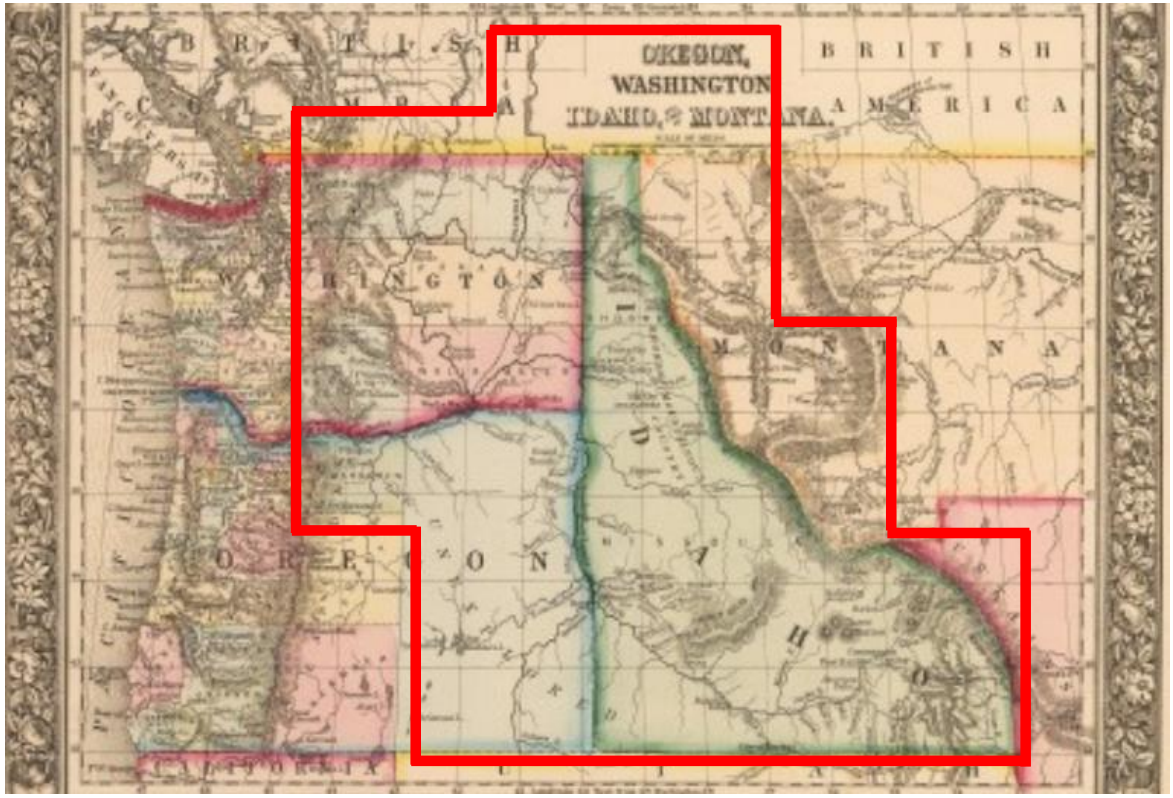
EXTENDING UNDERSTANDING OF AREA

Lesson 4: Extending Understanding of Area

Joseph is a young Nez Perce. He chose to learn about the land his ancestors lived in for a project at a school.

He found this old map in the town's library.

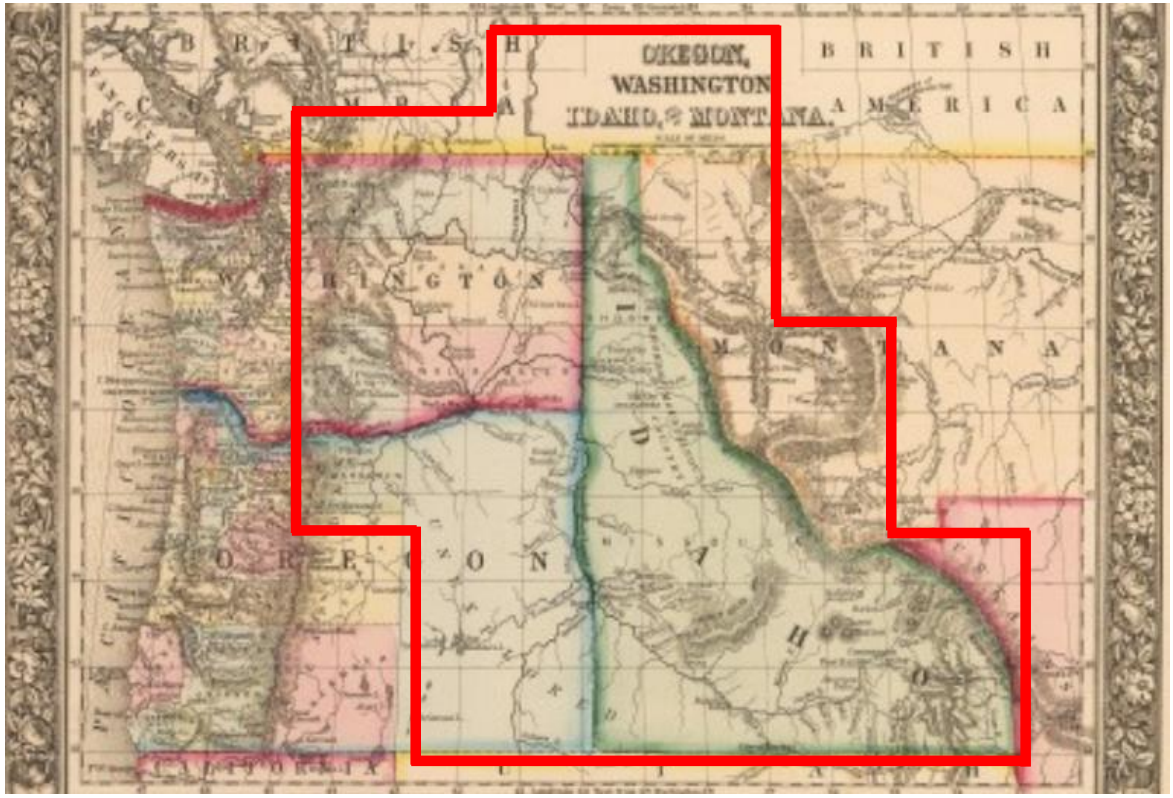
It shows the area of the United States the Nez Perce lived in before the states actually existed as states.



Lesson 4: Extending Understanding of Area

Unfortunately, when Joseph tried to find the area of the plan the Nez Perce lived in, he found some of the measurements were missing.

Let's explore ways to help Joseph **find the missing measurements** and then **calculate the area**.

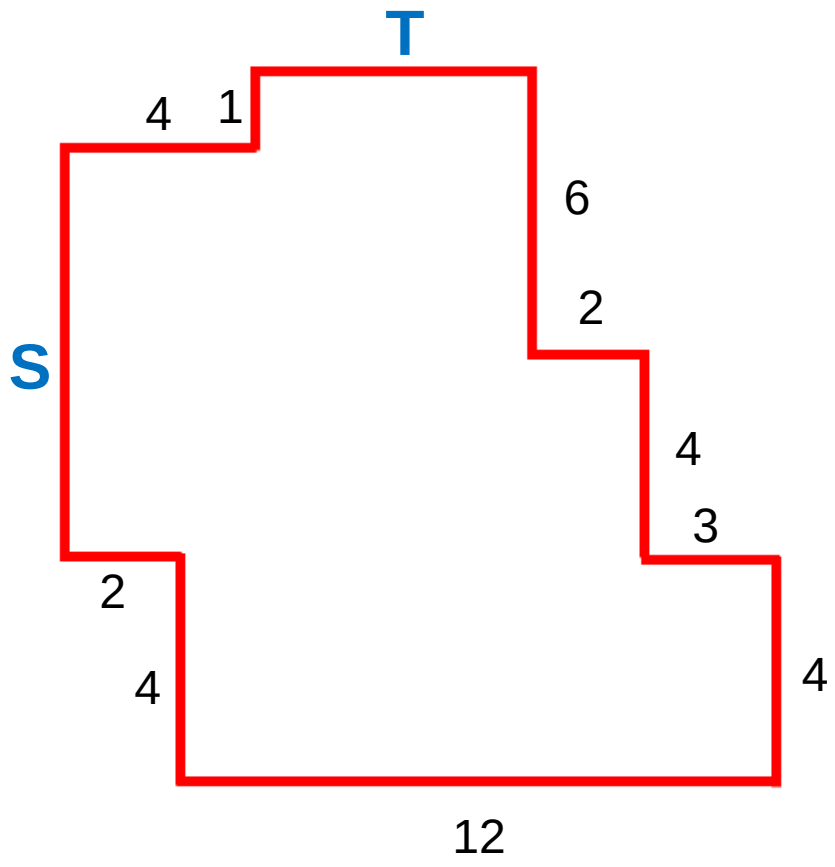


Lesson 4: Extending Understanding of Area

Here is small drawing of the Nez Perce living space. It was extremely large and measured many thousands of miles.

To help us solve the problem, the measurements have been changed to smaller numbers. This way we can work on the problem without working with such large numbers.

The missing measurements are labeled with letters.



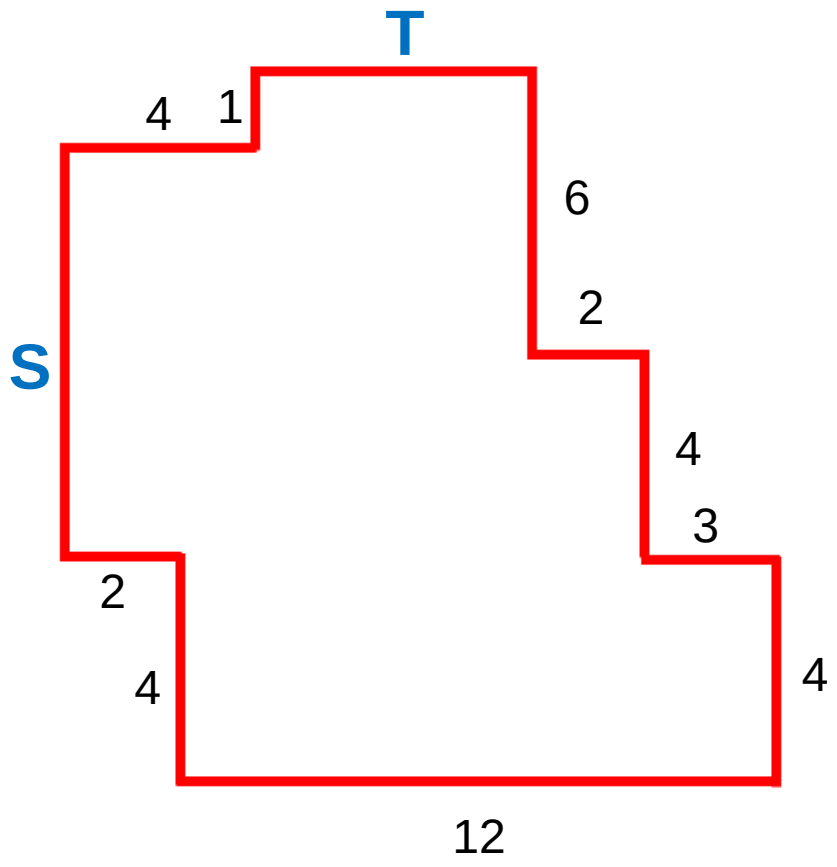
Lesson 4: Extending Understanding of Area

Work on your own for a few minutes to try to find the missing measurements.

Then, you will discuss your ideas with a partner.

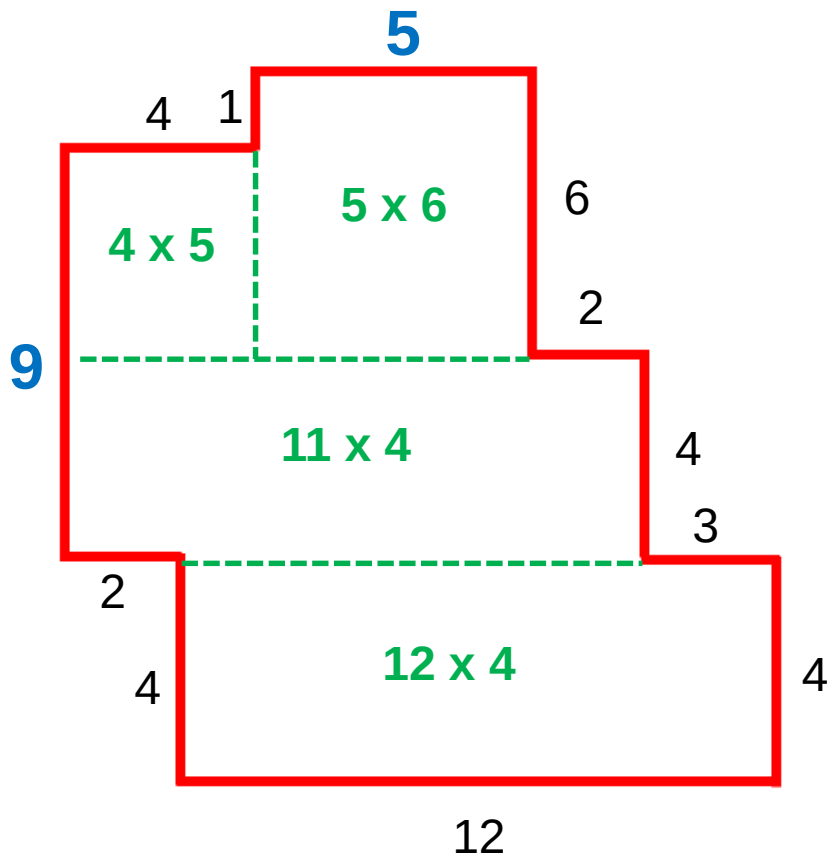
After you have the missing measurements, we will work on finding the area by decomposing the entire figure.

Finally, we will check our answers as class and decide how we could solve similar problems in the future.



Lesson 4: Extending Understanding of Area

EXAMPLE



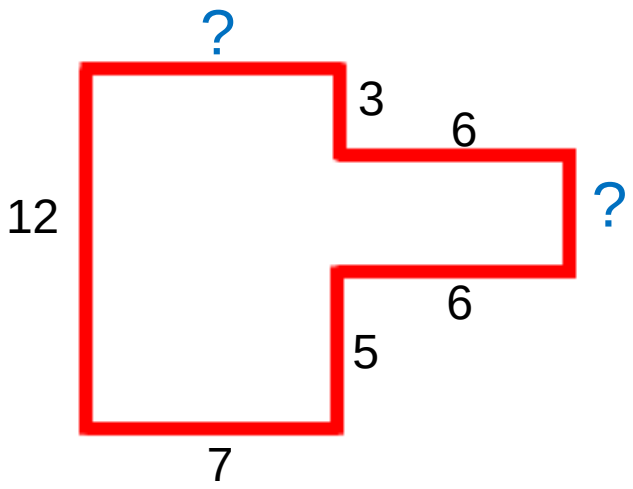
Lesson 4 Practice

Find the missing side length measurements.

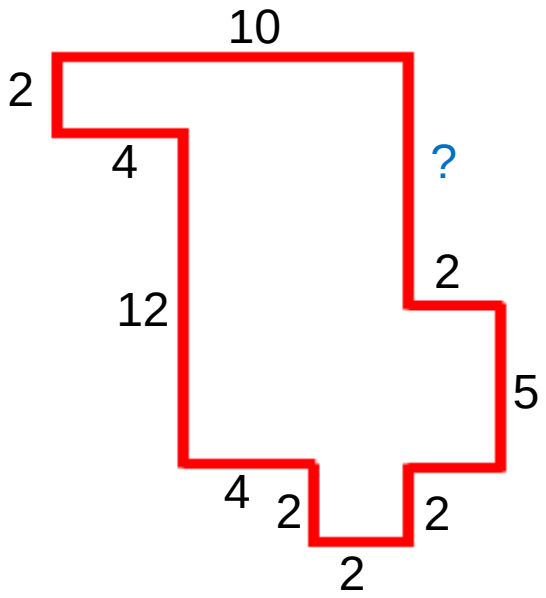
Decompose the shapes into rectangles.

Find the area in square units.

1.



2.



Lesson 5

EXAMINING ERRORS

Lesson 5: Examining Errors

Let’s explore some common errors students make when learning about area.

For each example, decide what the student’s error was.

Then, solve the problem correctly.

Finally, discuss with a partner how you could help the student understand the error and how to correct it. Use the Word Bank to help your explanations. You may also use models to help support your explanation.

Word Bank

decompose area iterate sides units

Lesson 5: Examining Errors

Student's Incorrect Answers

Lesson 6

REVIEW

Lesson 6: Review

This is a picture of a part of the Nez Perce living space called the “Palouse.”

The name comes from mispronunciations of local tribes’ names by French fur trappers who traveled through the area.

Tell a partner any reason you can think of that you might need to find the area of this beautiful land.

Lesson 6: Review

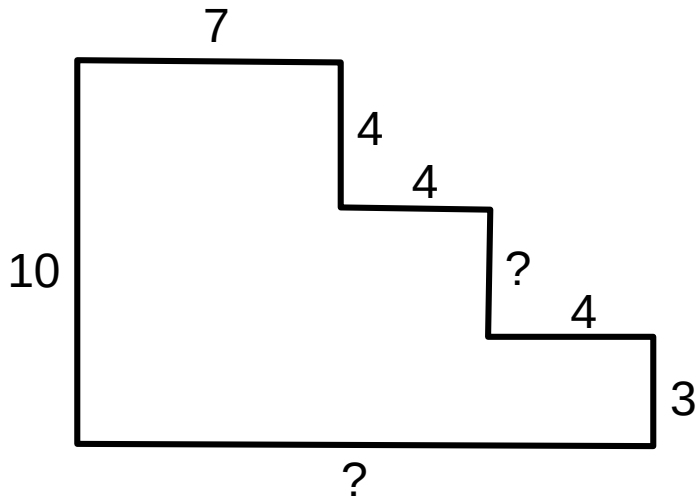
1. Find the missing side lengths.

2. What are the lengths of all of the sides added together?

3. Determine the area of the figure and write equations to describe the decomposing strategy you used.

4. Rick used a decomposing strategy, but thinks he did something wrong. Explain to him his mistake.

(7 x 4) + (4 x 10) + (4 x 3) = 28 + 40 + 12 = 80 square units.



Lesson 7

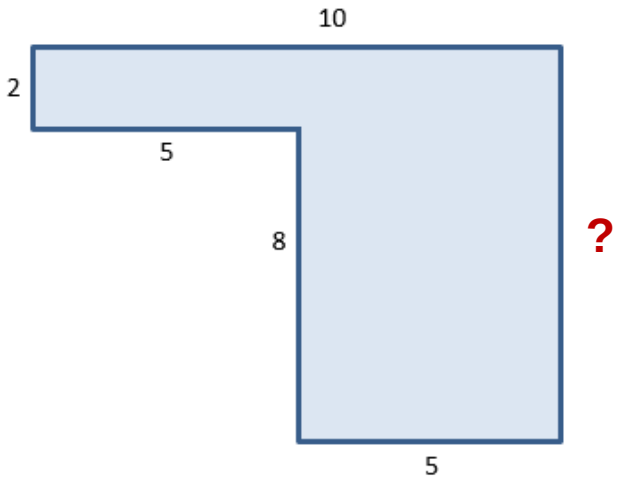
PRACTICE 2

Lesson 7: Practice Part 2

Complete the Lesson 7 Worksheet.

You will either need to find the area of a figure with a missing side length or, you will need to draw a figure with the given area that matches the given equations.

Name:_____

Figure	Area	Equations
1.  ?	A =	
2.	A = 72	(6 x 5) + (6 x 7) = 72

DMT INSTITUTE

"The Developing Mathematical Thinking Institute (DMTI) is dedicated to enhancing students' learning of mathematics by supporting educators in the implementation of research-based instructional strategies through high-quality professional development, curricular resources and assessments."

For more information contact
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