

DMT INSTITUTE

Developing Mathematical Thinking Institute (DMTI)



Professional
Development



Curricular
Resources



Assessment

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About the DMTI Modules

The DMTI modules are designed to guide classroom instruction and formative assessment for teachers implementing the DMTI curricular materials.

The lessons are not necessarily intended for a single day of instruction. Teachers are encouraged to use their professional judgement regarding pacing. A suggested number of weeks is provided.

DMTI Day Overview

Overall, each module highlights historical and/or cultural themes used to build the lessons. Each Day should start with a warm-up, one or two major components of a lesson, and a take-away.

Components of a DMTI DAY (whether 45, 60, or 90 minutes long)

Warmup (3-5 minutes)

Lesson Component – Problem Solving Situation

Lesson Component – Explanation of math concepts and ideas

Lesson Component – Varied Tasks

Lesson Component – Varied Practice

Takeaway (2-4 minutes)

DMTI Lesson Component Overview

Overall, each module highlights historical and/or cultural themes used to build the lessons. Each Lesson will focus on one or more of the following Lesson Components:

Lesson Component – Problem Solving Situation (~3 to 10 minutes)

Lesson Component – Explanation of Math Concepts and Ideas (~3 to 5 minutes; explanation of math concepts and ideas (with historically, culturally relevant and mathematically accurate ideas)

Lesson Component – Varied Tasks (~10-20 minutes; Completed together, in small groups or individually)

Lesson Component – Varied Practice (~15-30 minutes; Enactive, Iconic, Symbolic or Context, Iconic, and Symbolic)

Lesson Review (After every few lessons a review with different questions – skill, problem solving, conceptual, and justification – will be incorporated as both practice and a formative assessment or checkpoint for teachers.)

Grade 2

*NUMBER: PLACE VALUE WITH PART WHOLE AND
COMPARE PROBLEMS*

Module Sequence

Lesson 1: Part-Whole Situations

Lesson 2: Part-Whole: Practice

Lesson 3: Part-Whole: Writing Contexts

Lesson 4: Part-Whole: Iconic Models

Lesson 5: Solving Compare Situations:
Context

Lesson 6: Compare Situations: Practice

Lesson 7: Solving Compare Situations:
Iconic Models

Lesson 8: Solving Compare Situations:
Symbolic Models

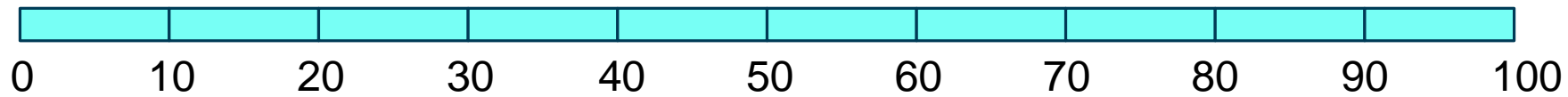
Lesson 9: Solving Compare Situations:
Making Models and Justification

Lesson 10: Compare Situations: Pocket
Survey

Lesson 11: Part-Whole and Compare
Situations: Summary and Varied Practice

Warmup

Using a unit of 10, count forward from 0 to 100 and then backward from 100 to 0.



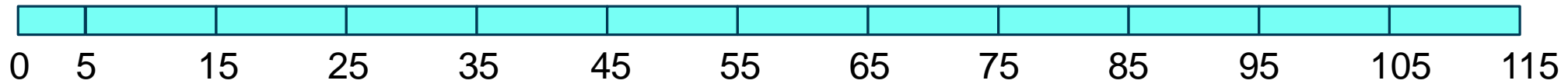
Warmup

Using a unit of 10, count forward from 0 to 100 and then backward from 100 to 0.

0

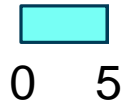
Warmup

Using a unit of 10, count forward from 5 to 115 and then backward to 0.



Warmup

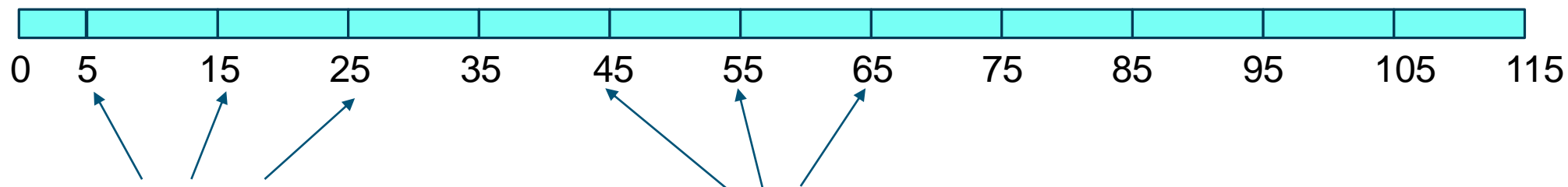
Using a unit of 10, count forward from 5 to 115 and then backward to 0.



Warmup

What patterns do you notice when counting by 10's?

What is staying the same and what is changing? Why?



The 5 ones stay the same.

The number of 10's keeps changing.

*We start with the 5 ones, and then never add more one units. So the **digit** 5 stays the same.*

We then add 1 unit of ten which gives us 1 ten with 5 ones or 15. We keep iterating a unit of 10, which gives us 2 tens, 3 tens and so on until we have 11 tens. The 11 tens with the 5 ones is 115.

Warmup

Practice counting with a partner.

- Count by 10's starting at 7 to 77 and back to 7.
- Count by 10's starting at 9 to 119 and back to 9.
- Count by 10's starting at 3 to 53 and back to 3.
- Count by 100's starting at 0 to 900 and back to 0.
- Count by 100's starting at 50 to 950 and back to 50.
- Count by 100's starting 35 to 735 and back to 35.

Warmup

1. What does **iterate** mean?
2. What stays the same when counting by 10's?
3. What changes when counting by 100's?
4. What is the relationship among units of 1, 10 and 100?

Lesson 1

PART-WHOLE SITUATIONS

Lesson 1: Part-Whole Situations

There are 3 second grade classrooms in the school. Each class has 27 students. How many second grade students are there altogether?

If there are 39 girls, how many boys are there?

1. Model both of these situations.
2. Write an equation for each.
3. Solve them using any method.

Lesson 1: Part-Whole Situations

There are 3 second grade classrooms in the school. Each class has 27 students. How many second grade students are there altogether?

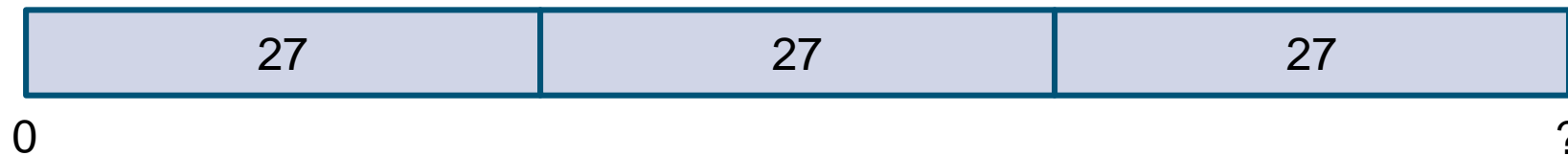
If there are 39 girls, how many boys are there?

This situation is called a **part-whole story problem**, because it consists of **two parts**, which are the **girls** and the **boys**, and then a **whole**, which is all of the **children**.

Lesson 1: Part-Whole Situations

Here is how to model the first situation. Take notes on your paper.

There are **3 second grade classrooms** in the school. **Each class has 27 students**. How many second grade students are there altogether?

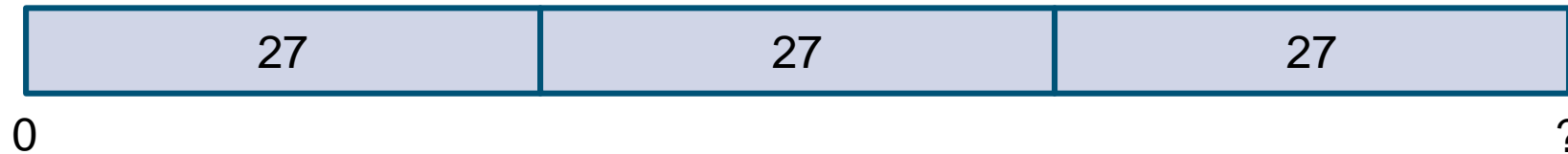


The equation to match this situation is $27 + 27 + 27 = ?$

Lesson 1: Part-Whole Situations

Here is how to model the first situation. Take notes on your paper.

There are **3 second grade classrooms** in the school. **Each class has 27 students.** How many second grade students are there altogether?



The equation to match this situation is $27 + 27 + 27 = ?$

How many students are their total?

Lesson 1: Part-Whole Situations

Tia's Method is called a Place Value Strategy. Take notes and see whether your strategy is similar.

$$27 + 27 + 27 = 20 + 20 + 20 + 7 + 7 + 7 \text{ (Decompose each 27 to 20 and 7)}$$

$$20 + 20 + 20 = 60 \text{ (Compose the 20's)}$$

$$7 + 7 + 7 = 21 \text{ (Compose the 7's)}$$

$$60 + 21 = 81 \text{ (Compose the 60 and 21)}$$

There are 81 students total.

Lesson 1: Part-Whole Situations

Paul's Method is called a Composing Strategy. Take notes and see whether your strategy is similar.

$$27 + 27 + 27 = 25 + 25 + 25 + 2 + 2 + 2 \text{ (Decompose each 27 to 25 and 2)}$$

$$25 + 25 + 25 = 75 \text{ (Compose the 25's)}$$

$$2 + 2 + 2 = 6 \text{ (Compose the 2's)}$$

$$75 + 6 = 81 \text{ (Compose the 75 and 6)}$$

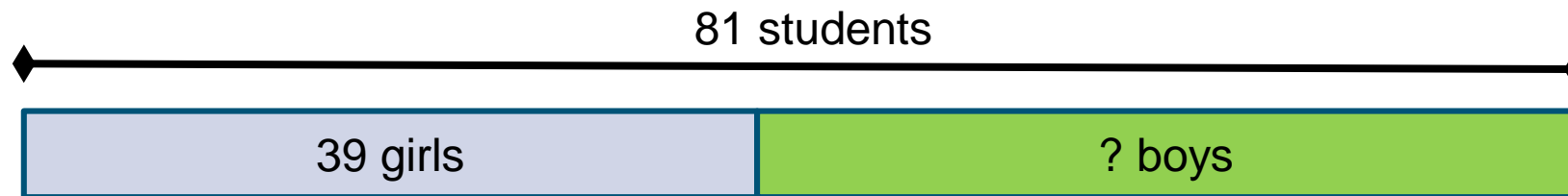
There are 81 students total.

Lesson 1: Part-Whole Situations

There are 3 second grade classrooms in the school. Each class has 27 students. How many second grade students are there altogether?

If there are 39 girls, how many boys are there?

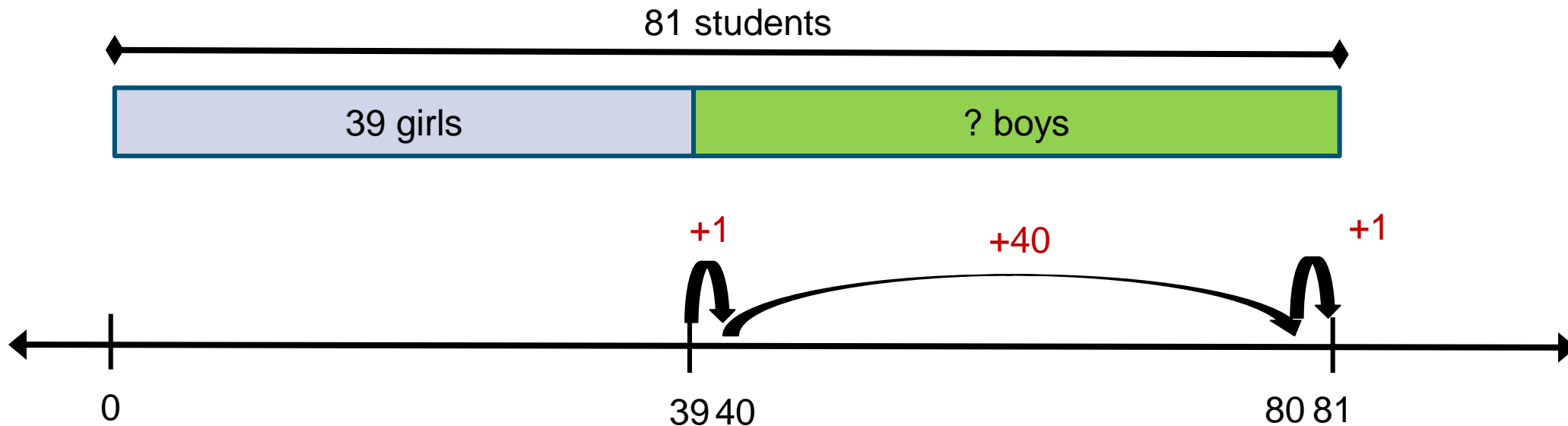
Here is how to model the second situation. Take notes on your paper.



The equation to match this situation is $39 + ? = 81$

Lesson 1: Part-Whole Situations

Now, let's solve the second problem. Here is Kie's method. (Take notes.)



$$1 + 40 + 1 = 42$$

42 boys

Describe Kie's method and why it works. Use Kie's way of thinking to solve this problem:

$$49 + ? = 95$$

Lesson 1: Review and Takeaways

4. How does the bar model help us solve story problems?
5. Solve this problem using the place value strategy? $38 + 23 = ?$
6. Solve this problem using the composing strategy? $17 + 35 = ?$

Lesson 2

PART-WHOLE SITUATIONS
PRACTICE

Lesson 2: Part-Whole Varied Practice

Here are some number sets to be placed back into our problem with classes, children, and girls and boys.

	Classes	Students	Girls	Boys
1.	3	30	?	40
2.	3	25	35	?
3.	3	28	51	?
4.	3	26	?	38
*5.	5	30	79	?

Model each situation, write equations, and then solve them.



Lesson 2: Part-Whole Varied Practice

In New Mexico, there are many white aster and purple columbine flowers. These flowers are shipped to stores across the United States. **Model and solve** for the total number of flowers packages in each situation.

6. In the first package there are 32 white asters and 32 purple columbines.
7. The second package had 75 white asters and 25 purple columbines.
8. A third package had 83 white asters and 54 purple columbines.
9. The fourth package had 29 white asters and 34 purple columbines.
- 10 The fifth package had 142 white asters and 173 purple columbines.

Lesson 2: Review and Takeaways

11. What is the most important math item you learned in this lesson.

12. With children, girls and boys, which are the parts and which are the whole?

Lesson 3

PART-WHOLE SITUATIONS
WRITING CONTEXTS

Lesson 3: Part-Whole Story Problems

Now it is your turn to write part-whole story problems. The situations and number choices are provided below.

1. Ants (54) – red (28) and black (?)
2. Roses (68) - red (?) and yellow (39)
3. Balloons ? – green (34) and orange (53)
4. Apples (116) – red (36) and green (?)
5. Animals (?) – cats (92) and dogs (58)

Model and solve these problems.

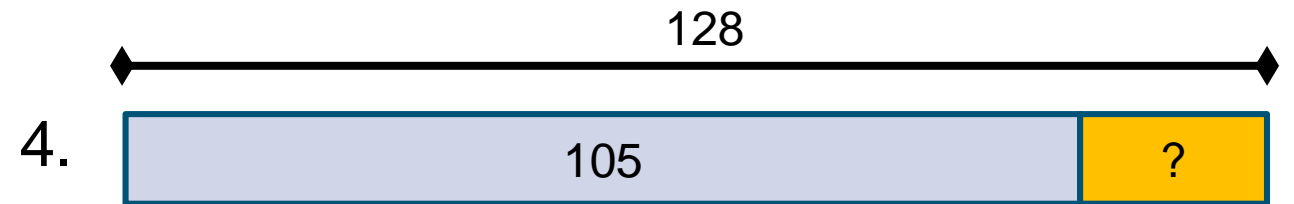
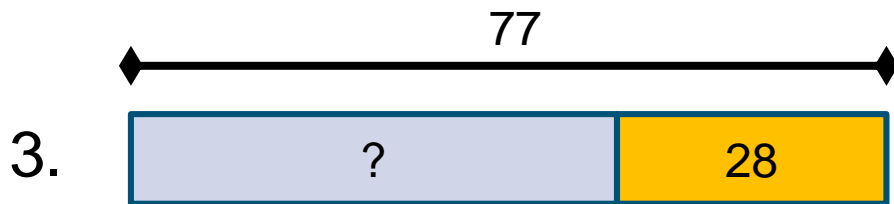
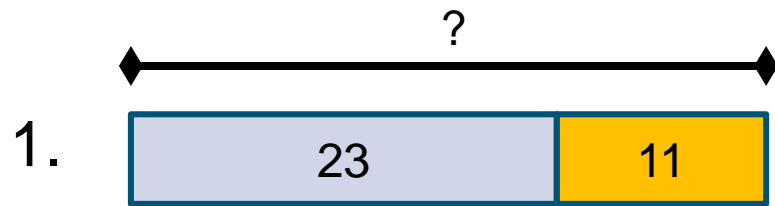
Lesson 4

PART-WHOLE SITUATIONS

ICONIC MODELS

Lesson 4: Iconic Part-Whole Models

For the following bar model diagrams, write a story problem, write an equation that matches the situation, and solve the situation.



Lesson 4: Review and Takeaways

5. What does it mean to have a part-whole situation?

6. If an ant hill had 74 red ants and 82 black ants, how many ants are on the ant hill?

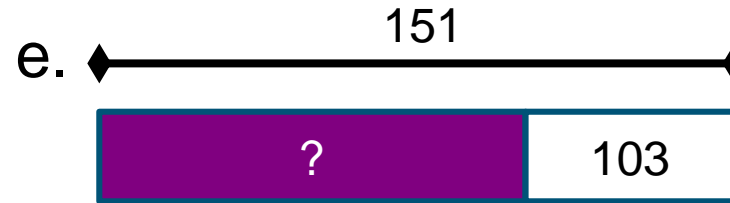
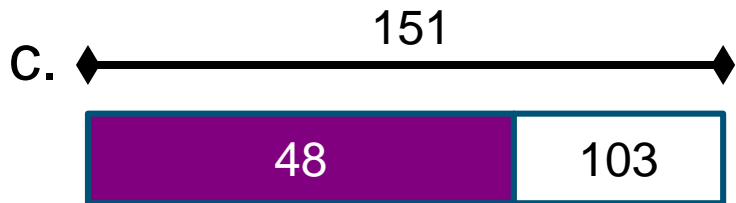
- a. Model the situation.
- b. Solve the situation.

Lesson 4: Review and Takeaways

7. Match the correct story problem to the correct bar model and equations.

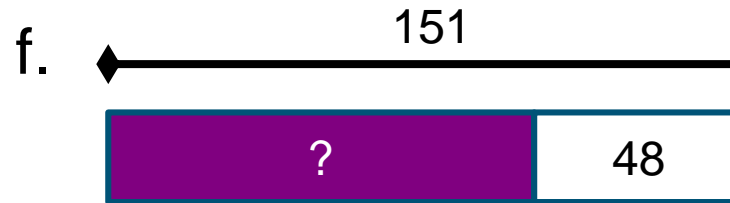
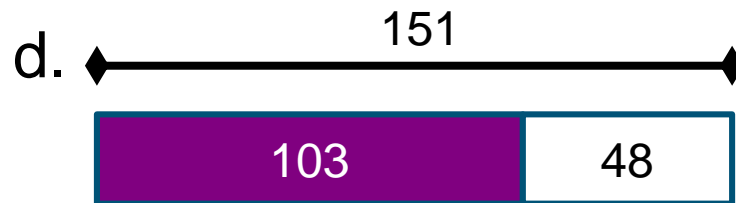
a. There are 151 flowers. Some are white and 48 are purple.

b. There are 103 white asters and 48 purple columbines. Altogether there are 151 flowers.



g. $151 = ? + 48$

h. $48 = 103 + 151$



i. $151 = 48 + 103$

j. $103 + 48 = 151$

Lesson 5

SOLVING COMPARE SITUATIONS



Lesson 5: Compare Situations

Tia and Frances planted a garden with corn. They picked 47 yellow ears of corn and 81 blue ears of corn. How many more blue ears of corn did they pick than yellow ears of corn?

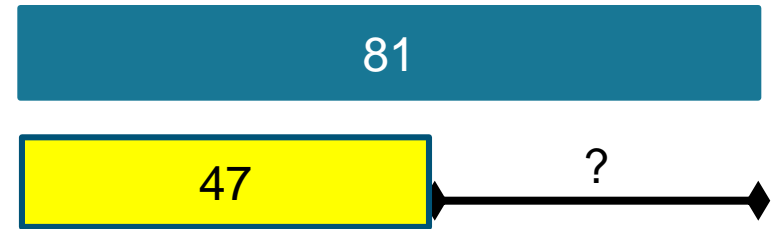
1. Model this situation.
2. Are there more yellow or blue ears of corn?
3. Write a number sentence for this situation.
4. Solve the problem.



Lesson 5: Compare Situations

Tia and Frances planted a garden with corn. They picked 47 yellow ears of corn and 81 blue ears of corn. How many more blue ears of corn did they pick than yellow ears of corn?

1. Model this situation.
2. Are there more yellow or blue ears of corn?



More blue ears.

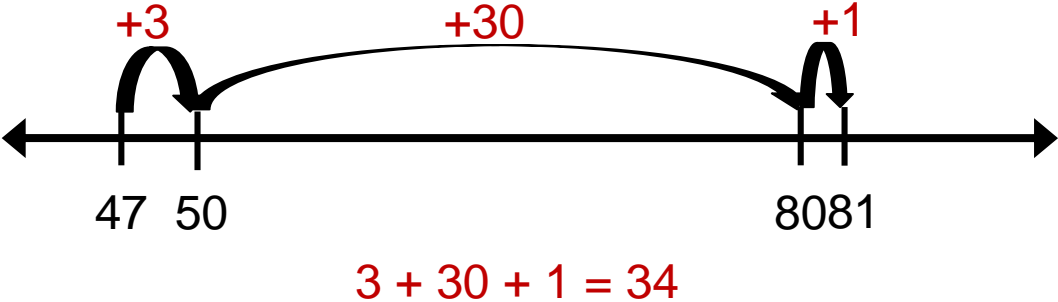
3. Write a number sentence for this situation. $47 + ? = 81$
4. Solve the problem. **34.**



Lesson 5: Compare Situations

Tia and Frances planted a garden with corn. They picked 47 yellow ears of corn and 81 blue ears of corn. How many more blue ears of corn did they pick than yellow ears of corn?

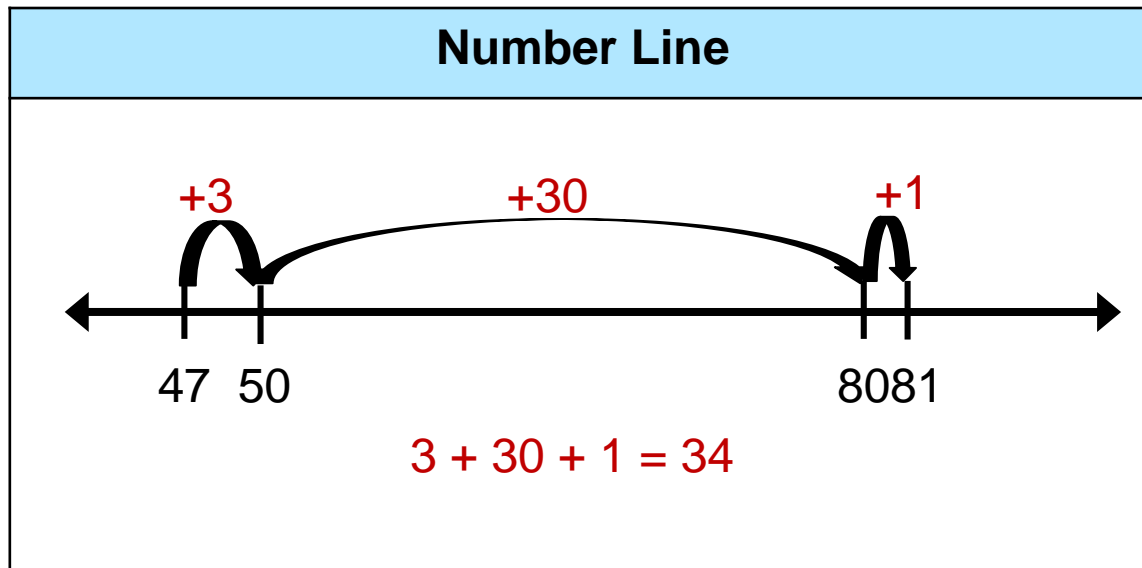
Let's examine three ways Tia solved the problem.

Number Line	Place Value	Compose with Friendly Numbers
 <p>$3 + 30 + 1 = 34$</p>	$81 - 47 = ?$ $81 - 40 = 41$ $41 - 1 = 40$ $40 - 6 = 34$	$47 + 3 = 50$ $50 + 30 = 80$ $80 + 1 = 81$ <p>So, $3 + 30 + 1 = 34$</p>

Word Bank
Unit Equation
Decompose
Compose

Lesson 5: Compare Situations

Explain how each of Tia's models work using the word bank to the right.



In the number line model, Tia added 3 to 47 to get 50. Then she added 30 to get to 80. Then she added 1 to get to 81. She then composed $3 + 30 + 1$ to get 34.

There are 34 more blue ears of corn than yellow ears of corn.

Word Bank
Unit Equation
Decompose
Compose

Lesson 5: Compare Situations

Explain how each of Tia's models work using the word bank to the right.

Place Value
$81 - 47 = ?$
$81 - 40 = 41$
$41 - 1 = 40$
$40 - 6 = 34$

In the place value model, Tia wanted to separate out 47 of the ears of corn to see how many are left. This would tell us how many more ears of blue corn there are than yellow ears of corns. She decomposed the 47 into a 40 and a 7. From 81 she took away 40 and got 41. Then she decomposed the 7 into a 1 and a 6. From 41 she took away 1 to get 40 and then took away the 6 to get 34.

There are 34 more blue ears of corn than yellow.

Word Bank
Unit Equation
Decompose
Compose

Lesson 5: Compare Situations

Explain how each of Tia's models work using the word bank to the right.

Compose with Friendly Numbers

$$47 + 3 = 50$$

$$50 + 30 = 80$$

$$80 + 1 = 81$$

$$\text{So, } 3 + 30 + 1 = 34$$

In the composing with friendly numbers model, Tia added 3 to 47 to get 50. Then she added 30 to get to 80. Then she added 1 to get to 81. She then composed $3 + 30 + 1$ to get 34.

There are 34 more blue ears of corn than yellow.



Lesson 5: Compare Situations

Tia and Frances planted a garden with corn. They picked 47 yellow ears of corn and 81 blue ears of corn. How many more blue ears of corn did they pick than yellow ears of corn?

This story is called a **compare problem**, because it asks us to **compare different groups**. In this situation we are comparing carrot seeds to pepper seeds. When we model this situation we use different bars or rectangles that start at the same 0.

Lesson 5: Review and Takeaways

5. What does it mean to have a part-whole situation?
6. Tia picked 72 yellow ears of corn and 53 blue ears of corn. How many more yellow ears of corn did she pick?
 - a. Model the situation.
 - b. Solve the situation with a number line strategy.
 - c. Solve the situation with a place value strategy.
 - d. Solve the situation with a composing strategy.
7. What is the most important thing you learned in this lesson?

Lesson 6

COMPARE SITUATIONS: VARIED PRACTICE WITH STORIES



Lesson 6: Varied Practice with Stories

Use the compare problem worksheet 6.1 to solve the following problems. Model each situation first, write an equation and then solve it using one of the methods listed.

Problems	Number Sets
1. Tia planted 26 green peppers and 46 red chilies. How many more red chilies did she plant than green peppers?	(38, 47) (125, 75) (184, 107)
2. Frances planted 75 green peppers and 55 red chilies. How many more green peppers did he plant than red chilies?	(84, 15) (134, 54) (163, 89)
3. Tia planted 65 green peppers. She planted 35 more red chilies than green peppers. How many red chilies did she plant?	(80, 25) (72, 29) (102, 17)
4. Frances planted 70 green peppers. He planted 52 fewer red chilies than green peppers. How many red chilies did he plant?	(143, 52) (185, 90) (162, 34)

Compare Problem Worksheet 6.1

Story Problem

Bar Model to represent the story.

Equation to represent the story

Solve

- **Number line**
- **Place value**
- **Compose friendly number**

Lesson 6: Varied Practice

Using the word bank, explain how you solved each of the four problems.

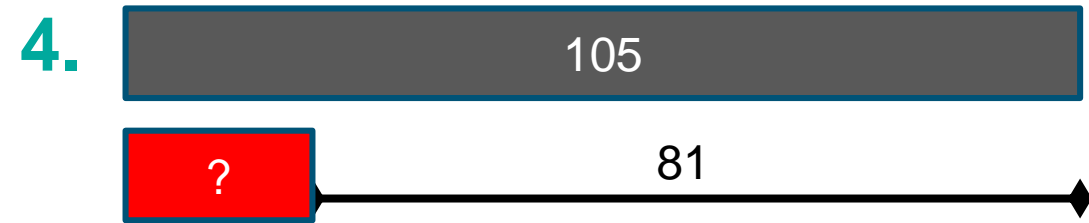
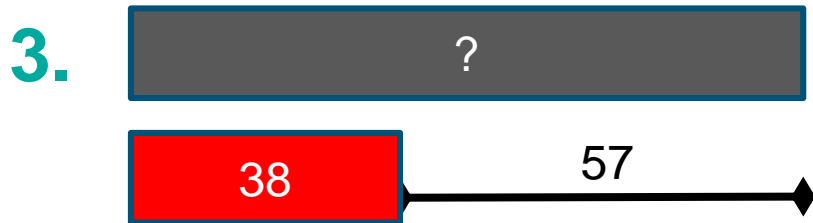
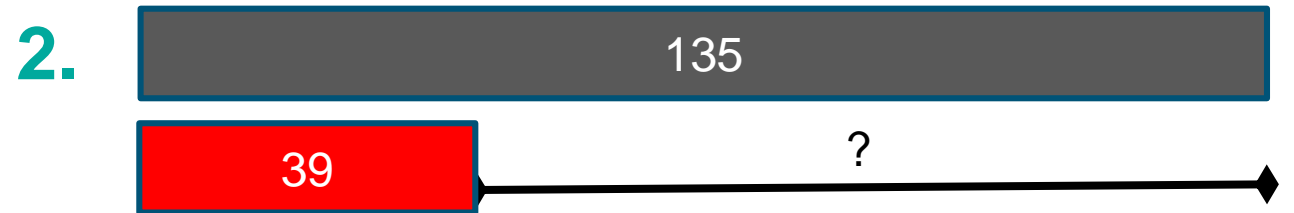
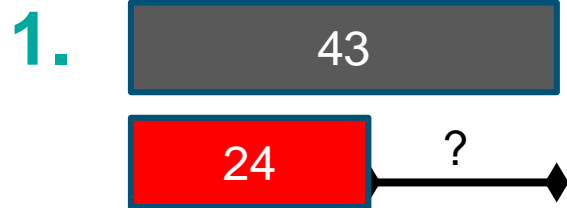
Problems	Explanation
1. Tia planted 26 green peppers and 46 red chilies. How many more red chilies did she plant than green peppers?	
2. Frances planted 75 green peppers and 55 red chilies. How many more green peppers did he plant than red chilies?	
3. Tia planted 65 green peppers. She planted 35 more red chilies than green peppers. How many red chilies did she plant?	
4. Frances planted 70 green peppers. He planted 52 fewer red chilies than green peppers. How many red chilies did he plant?	

Lesson 7

SOLVING COMPARE SITUATIONS: ICONIC MODELS

Lesson 7: Iconic Models

For the following bar model diagrams, write a story problem that compares the number of red to black ants, write an equation that matches the situation, and solve the problem. (Use Practice Mat 8.1).



Compare Situations: Practice Mat 7.1

Number Sentence:

Story:

Bar Model:

Solve:

Lesson 7: Review and Takeaways

5. What was the most difficult part of today's work?

6. What was the easiest part of today's work?

Warmup: Quick Draws

Draw this model.

If this is a model of 4, draw a model of what 8 would look like.



Use the terms *partition*, *iterate*, *compose*, *decompose* and *unit* to describe your drawing strategies.

Warmup: Quick Draws

Draw this model.

If this is a model of 12, draw a model of what 3 would look like.



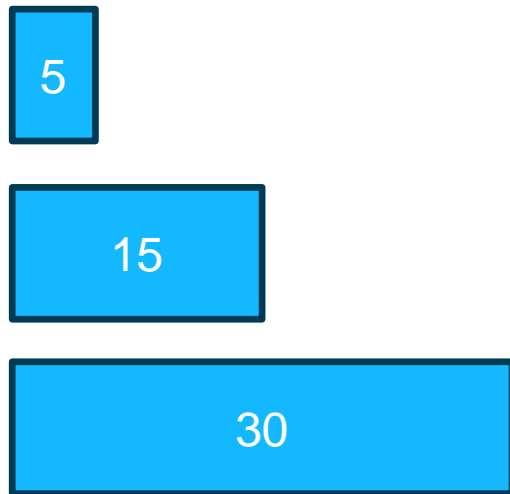
Use the terms *partition*, *iterate*, *compose*, *decompose* and *unit* to describe your drawing strategies.

Warmup: Quick Draws

Draw this model.

If this is a model of 5, draw a model of what 15 would look like.

What would 30 look like?



Use the terms *partition*, *iterate*, *compose*, *decompose* and *unit* to describe your drawing strategies.

Lesson 8

COMPARE SITUATIONS: SYMBOLIC MODELS

Lesson 8: Compare Situations – Symbolic Models

For the following equations, create a bar model to represent a compare situation, write a story problem, and solve the situation. (Use Practice Mat 8.1)

1. $27 + ? = 53$ (Story situation: green and red chilies)
2. $? + 61 = 115$ (Story situation: red and black ants)
3. $79 + 85 = ?$ (Story situation: girls and boys)

Compare Situations: Practice Mat 8.1

Number Sentence:

Story:

Bar Model:

Solve:

Lesson 8: Review and Takeaways

4. For a compare problem, Kevin is wondering which of the following story problems is correct. Explain which story fits a compare problem and which one does not.

$$28 + ? = 59,$$

A. There are 28 red ants. There are 59 black ants. How many more black ants are there than red ants?

B. There are 59 ants. There are 28 red ants and some black ants. How many black ants are there.

5. What was your favorite thing to do in math today?

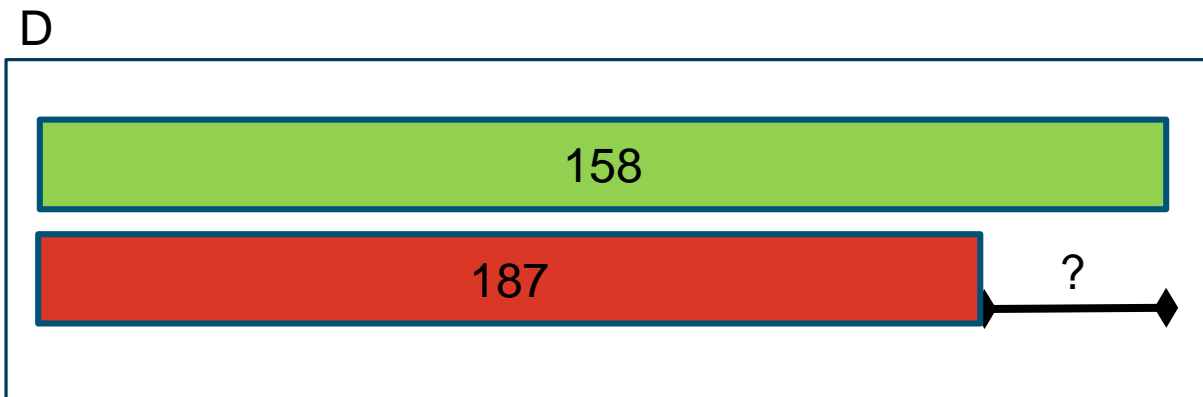
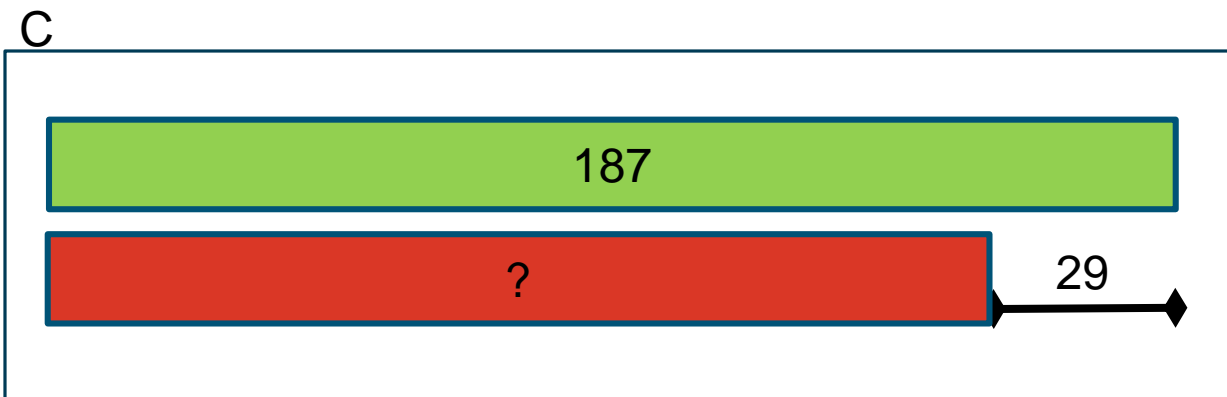
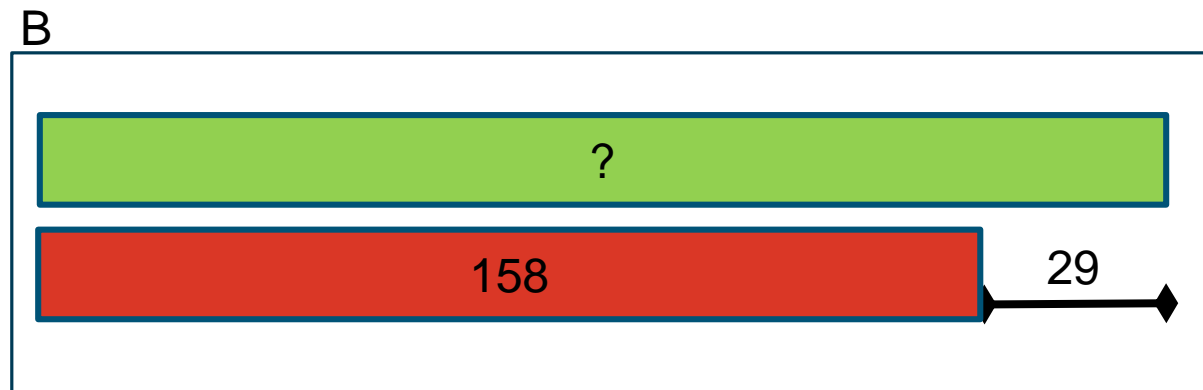
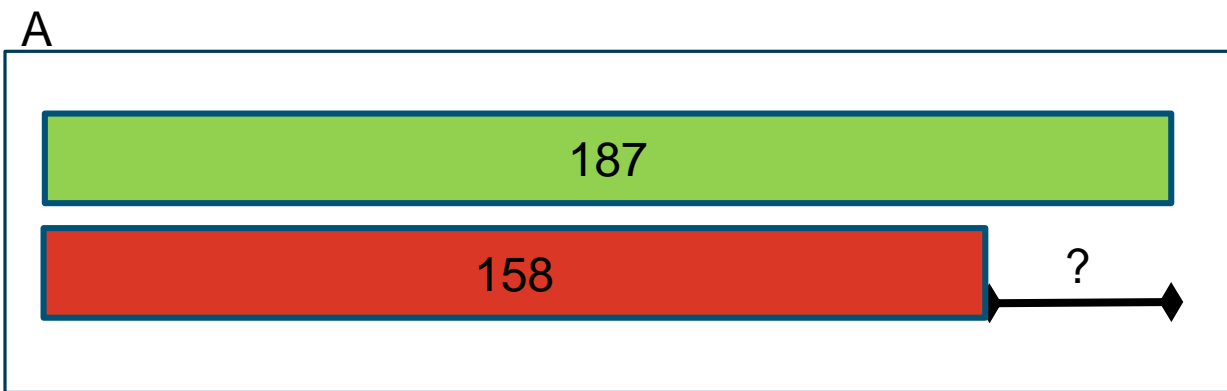
Lesson 9

SOLVING COMPARE SITUATIONS: MATCHING MODELS AND JUSTIFICATION



Lesson 9: Model Matching

1. Match the following compare models – bar model, equation and story problem.





Lesson 9: Model Matching

E. $158 = 187 + ?$

F. $187 = ? + 29$

G. $158 + 29 = ?$

H. $158 + ? = 187$

I There are 187 peppers in the garden. Some of them are green peppers and 29 of them are red chilies. How many of the plants are green peppers?

J There are 158 red chilies in the garden. There are 29 more green peppers than red chilies. How many green peppers are there in the garden?

K There are 158 red chilies in the garden. There are 187 green peppers. How many more green peppers are there than red chilies?

L There are 187 green peppers. There are 29 more green peppers than red chilies. How many red chilies are there in the garden?

Word Bank

Unit Compare
Decompose Part-Whole
Compose Equation

Lesson 9: Model Matching

Tia and Frances are wondering about two stories and whether they both could match the same number sentence below:

$$135 = ? + 40$$

2. Justify, using the word wall, why both might be correct.

There are 135 red apples.
There are some green apples.
And we know there are 40 more red apples than green apples. How many green apples are there?

There are 135 apples.
Some of them are red apples and 40 of them are green apples. How many are red apples?

Lesson 9: Review and Takeaways

3. What is a compare problem?

4. Draw a bar model to represent “I have 32 peppers. 8 are green peppers and the rest are red peppers. How many red peppers are there?”

5. What math idea from this lesson is most important to remember?

Lesson 10

COMPARE SITUATIONS: POCKET SURVEY



Lesson 10: Pocket Survey

We are going to conduct a survey to find out how many pockets each person has in the class. Work in groups of three. Your group needs to create a plan to count the number of pockets of 10 students in the class. Complete the following:

1. Create a graph of the number of pockets for your sample of 10 students.
2. What is the greatest number of pockets a person has? What is the least number of pockets?
3. How many pockets does everyone have in your sample?
4. Write one part-whole story and write one compare story with your pocket data.



Lesson 10: Pocket Survey

Have each group stand up and walk around the room to look at each group's representation of the number of pockets. Ask the following questions:

5. Who in the class had the most pockets?
6. Who in the class had the least pockets?
7. If another student walked into the class, how many pockets do you think the student will have?
8. Why did you choose this number?

Lesson 11

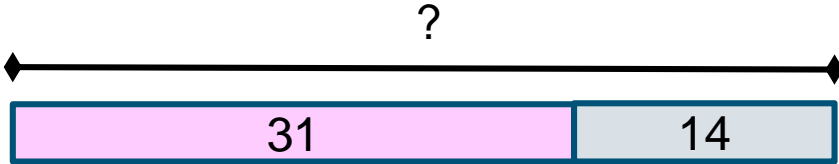
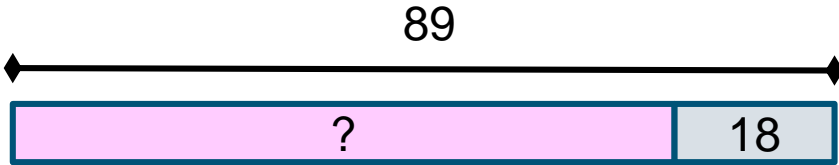
PART-WHOLE AND COMPARE SITUATIONS
SUMMARY AND VARIED PRACTICE

Lesson 11: Part-Whole and Compare

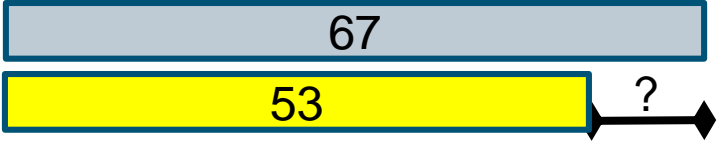
Situation: 87 red ants and 74 black ants.

1. Write a part-whole story problem about the ants.
2. Write a compare story problem about the ants.
3. Create a bar model to represent each story.
4. Solve the problems.
5. Explain what the difference is between a part-whole story and a compare story.
6. Pick one of your methods for solving the problem and explain how you solved it using the words unit, decompose, and compose.

Lesson 11: Part-Whole Situations

Story Problem	Bar Model	Equation	How I solved the problem...
There were 28 children swimming in the pool. 18 of the children were girls. How many were boys?			
		$136 + 19 = ?$	
			

Lesson 11: Compare Situations

Story Problem	Bar Model	Equation	How I solved the problem...
A farmer has 38 red chilies and 53 peppers. How many more peppers does the farmer have than red chilies?			
		$36 - 19 =$	
*A farmer has a farm with 84 chickens and pigs in all. There are 32 more chickens than pigs. How many chickens and pigs are on the farm?			

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