# DMT (M) INSTITUTE

#### Developing Mathematical Thinking Institute (DMTI)



Professional Development



Curricular Resources



Assessment

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### About the DMTI Modules

The DMTI modules are designed to guide classroom instruction and formative assessment for teachers implementing the DMTI curricular materials.

The lessons are not necessarily intended for a single day of instruction. Teachers are encouraged to use their professional judgement regarding pacing. A suggested number of weeks is provided.

# **DMTI Day Overview**

Overall, each module highlights historical and/or cultural themes used to build the lessons. Each Day should start with a warm-up, one or two major components of a lesson, and a take-away.

#### Components of a DMTI DAY (whether 45, 60, or 90 minutes long)

Warmup (3-5 minutes)

Lesson Component – Problem Solving Situation

Lesson Component – Explanation of math concepts and ideas

Lesson Component – Varied Tasks

Lesson Component – Varied Practice

Takeaway (2-4 minutes)

# Grade 1

### LINEAR MEASUREMENT AND PLACE VALUE 2-3 WEEKS

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### Module Sequence

Lesson 1: How Many Steps?

**Lesson 2:** Measuring Lengths with Cubes and Paper Clips

Lesson 3: Paper Strips

Lesson 4: Building a 10 Unit Ruler

**Lesson 5:** Practicing Length Measurements with a Ruler

Lesson 6: Broken Rulers

Lesson 7: Comparing Lengths Part 1

**Lesson 8:** Comparing Lengths Part 2

Lesson 9: Examining Errors Part 1

**Lesson 10:** Measuring Lengths in Units of 10 and Units of 1 Part 1

**Lesson 11:** Measuring Lengths in Units of 10 and Units of 1 Part 2

**Lesson 12:** Comparing Lengths Using Units of 10 and Units of 1

**Lesson 13:** Examining Errors Part 2

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# Lesson 1

#### HOW MANY STEPS?

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### Warm Up: Counting Using Hand Measurements

1. Place your hands or index fingers together. Ask students to think of a number that could be used to name how far apart your fingers/hands are if there is no space between them. (Answer: 0)

2. Have students count aloud and move their fingers/hands in unison with you as you count to 10 and show the corresponding lengths with the space between your fingers/hands.

3. Then count backwards from 10 to 0 following the same process.

4. Finally, have students show certain lengths with their fingers/hands (e.g. "Show me 7. Show me 4. What is 1 more than 4? Yes, 5. Now show me 6. Show me 7. Show me 8. What comes next? Yes, 9. Show me 10. Now show me 5. Now 10. Now 5. Now 4, 5, 6, 4, 5, 6, 7, 8, 9, 10, 5, 0."



In the past, before we had tools to measure with, people often measured using their steps.

A person would walk from one place to another and count the steps it took. They sometimes called the steps "paces." The number of paces was the length between the two places.



If you were standing in this doorway, how many steps do you think it would take to walk to this other doorway? Tell a partner your thinking and then be ready to share your thinking with the class.

Tanya was on a field trip visiting an historical site in Jemez Springs.

Her teacher told a story of how she had visited this place as a young girl and had played with her classmates in the pathway by running back and forth between the walls.

Tanya decided to test the **length** between the walls by counting her steps. She took the biggest steps she could.

Let's count her steps.



Tanya found it took 8 of her biggest steps to walk across the path.

Tanya's friend Max measured the length between the walls with his steps.

It took Max 12 steps.

Tanya and Max couldn't understand why they had different numbers of steps.

1. Why do you think they had different numbers of steps?

2. Use your own steps to show your thinking.



Let's practice measuring lengths with our steps just like Tanya and Max.

You will work with a partner to count your steps as you walk between the different parts of our room listed below.

One partner walks and the other partner counts each time the walking partner's foot touches the ground. Take turns so each partner gets to measure her or his steps.

Take medium-sized steps.

- A) Desk to door
- B) Wall to wall
- C) Teacher to whiteboard

D) Bookshelf to chair

Did you and your partner have the same problem Tanya and Max did or were your measurements the same? Why?

# Lesson 2

### INFORMAL LINEAR MEASUREMENT

MEASURING LENGTHS WITH CUBES AND PAPER CLIPS

We are going to learn how to use cubes to find out how long and tall different things in the room are.

This means we are going to learn to *measure* how long and tall these objects are.

We are also going to make a stick of connecting cubes. Try to follow the cubes I show you by placing them in front of you and connect them together.

We will count the cubes to see how long the stick is.

Just like when we count, we need to think about what number to start with.

"What number do we use to tell us we have not counted or measured anything yet?"

"We say 0 to mean nothing has been counted."

Let's start making a stick with connecting cubes. We will use these cube sticks to measure objects in the room later.

How many cubes are there?



What if we add one more?

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### Now how many cubes are there?



#### What if we add one more?

*"When you copy a unit over and over again with no gaps or overlaps, this is called iterating the unit. Iterating is copying the unit."* 

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#### Let's start with 0.

Let's count how long the cube stick is.



"The length of the cube stick is 5 cubes."

"We iterated the unit cube 5 times."

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Place your hand on the floor like this.

Take the cube stick and place it next to your hand.

Find your *longest* finger. Your longest finger might be different than the picture.

Compare the *length* of your hand *(from your wrist to the tip of your longest finger)* to the length of the 5 cube stick.



#### In the picture:

Which is *longer*?

Which is *shorter*?

Or, are they the same *length*?

What about your hand?

*"If the length of your hand was <u>longer</u> than the cube stick, raise your hand."* 

Place your hand on the floor like this.

Take the cube stick and place it next to your hand.

Find your *longest* finger. Your longest finger might be different than the picture.

Compare the *length* of your hand (from your wrist to the tip of your longest finger) to the length of the 5 cube stick.



#### In the picture:

Which is *longer*?

Which is *shorter*?

Or, are they the same *length*?

What about your hand?

*"If the length of your hand was <u>shorter</u> than the cube stick, raise your hand."* 

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Place your hand on the floor like this.

Take the cube stick and place it next to your hand.

Find your *longest* finger. Your longest finger might be different than the picture.

Compare the *length* of your hand (from your wrist to the tip of your longest finger) to the length of the 5 cube stick.



#### In the picture:

Which is *longer*?

Which is *shorter*?

Or, are they the same *length*?

What about your hand?

*"If the length of your hand and the cube stick were the <u>same</u> length, raise your hand."* 

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Now we will use a different *unit* to measure.

We will use *paper clips* as the unit.

You will measure the length of your hand by carefully *iterating* the paper clips. Let's look at the example.



1 paper clip



"The hand in the picture has a length of 3 paper clips." Why is the length 3 paper clips but 5 cubes? *"The measurements are different because the paper clips are"* longer than the edge of the cubes. It takes fewer paper clips to measure the same length."

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Now let's practice measuring other things around the room using cubes and then paper clips as the *units*.

- 1. You will work with a partner.
- 2. Using the pictures on the worksheet, find the *real* objects in the room.

3. Use your cubes to measure the lengths (or heights) of the *real* objects you found. Write this number.

4. Then use paper clips to measure the length of the same object and write this number.

5. Follow the directions on the worksheet and circle which object was *taller*, *longer*, *shorter* or *smaller*.

Lesson 2 Worksheet: Measuring Lengths with Cubes and Paper Clips



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# Lesson 3

### INFORMAL LINEAR MEASUREMENT PAPER STRIPS

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### Lesson 3: Paper Strips

We are going to measure the length of objects around the room as well as parts of the room itself using new *units*.

Instead of cubes and paper clips, we will use paper strips of different lengths.

You will work with a partner.

Let's first sort the paper strips from *longest* to *shortest*.



### Lesson 3: Paper Strips

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Instead of cubes and paper clips, we will use paper strips of different lengths.

You will work with a partner.

Let's first sort the paper strips from *longest* to *shortest*.



### Lesson 3: Paper Strips

Before you actually take each measurement, you will *predict* how many of each paper strip unit the measurement will be. Discuss your predictions with your partner.

Then you and your partner will work together to make all of the measurements on your worksheet.

Make sure each partner gets to use the paper strips, make a prediction, and also writes the numbers for the measurements.

You can see line segments that will help show you which part of the objects you need to measure.

If your measurement is only part of a paper strip, decide if you want to count that part as one whole unit or if you want to not include that unit.

Lesson 3 Worksheet: Paper Strips

Name(s):								
Measure this	Red	Green	Yellow	Measure this	Red	Green	Yellow	
Student's desk				<b>Longest</b> wall in the room				
Teacher's desk				Shortest wall in the room				
Chair				Doorway				

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# Lesson 4

### INFORMAL LINEAR MEASUREMENT BUILDING A 10 UNIT RULER

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We are going to build a measurement tool called a *ruler*.

We use rulers to measure the length of objects.

Our rulers will be 10 units so that we can practice what we have learned about place value as we measure.

Let's start at the left side of the background paper and glue one colored *unit* to the background.



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Now glue another *unit* using a different color of paper next to the first unit. This means you just *iterated* (copied) a unit of length.



When you iterate units of length, you want to make sure you do not have any gaps or overlaps.

Continue gluing new units of length by going back and forth between colors until you've used all 10 units.



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After we have glued all 10 units down to the background paper, we need to label the ruler. We will label the count of units at the end of each complete unit.



What number do we use to tell us we have not counted or measured anything yet?

Let's try using our new ruler to measure different objects' lengths.

Remember, we are measuring in units.

You will need to write the number of units you found each object measured.

You will be asked to measure different parts of each object in the units on your ruler.

Lesson 4 Worksheet: Using a 10 Unit Ruler

Name:						
Measure this	Measure this	Measure this				
How <i>tall</i> was the desk? units	How <i>tall</i> was the bookcase?	How <i>tall</i> was the trash can?				
	units	units				
How <b>wide</b> was the desk? units	How <i>wide</i> was the bookcase?	How <i>wide</i> was the trash can?				
	units	units				
# Lesson 5

### INFORMAL LINEAR MEASUREMENT PRACTICING LENGTH MEASUREMENTS WITH A RULER



## Lesson 5: Length Measurements

We are going to continue practicing using our 10 unit ruler.

Measure each picture and write the number of *length units* you found each picture was.

Then, cut out the pictures and sort them from *shortest* to *longest*.

For any pictures that are the same length, order them however you like.



\_units





\_units



\_\_\_\_units



\_units

# Lesson 6

### LINEAR MEASUREMENT

### BROKEN RULERS

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Some students were using rulers to measure the length of different objects.

Unfortunately, the rulers were broken and didn't start at 0 and end at 10.

Help the students understand which measurement is correct.

You will need to explain the incorrect measurements so that the students know how to avoid their mistakes in the future.

Monica says the snake is 6 units long.

1) Is Monica correct?

2) Explain why or why not.

## Why did Monica think the snake was 6 units long?



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Martha says the pencil is 5 units long.

1) Is Martha correct?

2) Explain why or why not.

Why did Martha think the pencil was 5 units long?



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Bill says the marker is 7 units long.

- 1) Is Bill correct?
- 2) Explain why or why not.

## Why did Bill think the marker was 7 units long?



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Isabel says the pencil is 3 units long.

1) Is Isabel correct?

2) Explain why or why not.



Why is Isabel's measurement of 3 units correct?

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Steven says the pencil is 4 units long.

1) Is Steven correct?

2) Explain why or why not.

Why did Steven think the pencil was 4 units long?



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Jamelle says the snake is 5 units long.

1) Is Jamelle correct?

2) Explain why or why not.

Why is Jamelle's measurement of 5 units correct?



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Miguel says the marker is 6 units long.

1) Is Miguel correct?

2) Explain why or why not.

## Why did Miguel think the marker was 6 units long?



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Pirna says the marker is 5 units long.

1) Is Pirna correct?

2) Explain why or why not.

Why is Pirna's measurement of 5 units correct?



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## Lesson 7

### LINEAR MEASUREMENT COMPARING LENGTHS PART 1

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We are going to use our 10 unit ruler to measure the lengths of two different objects and then *compare* them.

When we *compare*, we find how much *longer* or *shorter* one object is than the other.

Let's look at some examples together.

How tall is Person A?

How tall is Person B?

Which person is *taller*?

How much *taller*?

Person A is 9 units tall. Person B is 7 units tall. Person A is <u>taller</u>.

Person A is 2 units <u>taller</u> than Person B.

This means Person B has to grow 2 units <u>taller</u> to be just as tall as Person A.



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How long is the blue rod?

How long is the green rod?

Which rod is longer?

How much *longer*?

The blue rod is 5 units long.

The green rod is 8 units long.

The green rod is longer.

The green rod is 3This means the blue rod needsunits longer than theto be 3 units longer to be just asblue rod.long as the green rod.



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Find each of the real objects shown in the pictures in our classroom.

Use your 10 Unit Ruler to measure how long or tall each object is. The gray segments will help you know whether you are measuring how long something is or how tall something is.

Write a number for each measurement.

For each pair, *circle* the object that is *taller* or *longer*. If both objects are the *same length*, circle both.

Then, write a number to show how much taller or longer that object is than the other object. If both objects are the same length, write the number 0 to show that they have nothing different about their lengths. Lesson 7 Worksheet: Comparing Lengths Part 1



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# Lesson 8

### LINEAR MEASUREMENT COMPARING LENGTHS PART 2

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## Lesson 8: Comparing Lengths Part 2

In our last lesson we measured objects in the room and decided which was longer or taller and also how much longer or taller.

Now we will do the same thing with some pictures.

Since we are measuring the length of pictures and not real objects, you need to draw in markings that show the units of length.

You will also write numbers that show how you counted the length measurements.

Sometimes you will be asked to circle the *longer* picture. Sometimes you will be asked to draw a box around the *shorter* picture. If the pictures are the same, mark both like we have done before.

The first example has been done for you.

#### Lesson 8 Worksheet: Comparing Lengths Part 2 Page 1



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#### Lesson 8 Worksheet: Comparing Lengths Part 2 Page 2



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# Lesson 9

### LINEAR MEASUREMENT EXAMINING ERRORS PART 1

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Now that we are very good at measuring, we are going to help some other students learn how to fix their mistakes.

For each example:

- 1) Think about what the student's mistake might be.
- 2) Tell a partner what you think the mistake was.

3) Discuss with a partner how you could help each student correct his/her mistake.

Think about what the student's mistake might be.
Tell a partner what you think the mistake was.
Discuss with a partner how you could help each student correct his/her mistake.

Oliver has measured the marker this way and says that it is 4 cubes long.



Think about what the student's mistake might be.
Tell a partner what you think the mistake was.
Discuss with a partner how you could help each student correct his/her mistake.

Sandra thinks that the picture of the tree is as tall as 6 cubes



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Think about what the student's mistake might be.
Tell a partner what you think the mistake was.
Discuss with a partner how you could help each student correct his/her mistake.

Michele says the length of the pencil is 7 units.



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Think about what the student's mistake might be.
Tell a partner what you think the mistake was.
Discuss with a partner how you could help each student correct his/her mistake.

Oscar says the length of the pencil is 8 units. He has shown how he counted units by making marks.



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Think about what the student's mistake might be.
Tell a partner what you think the mistake was.
Discuss with a partner how you could help each student correct his/her mistake.

Emily made a ruler just like we did. She then wrote numbers on her ruler.



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Think about what the student's mistake might be.
Tell a partner what you think the mistake was.
Discuss with a partner how you could help each student correct his/her mistake.

Jeff says the green rod is longer than the yellow rod.

Think about what the student's mistake might be.
Tell a partner what you think the mistake was.
Discuss with a partner how you could help each student correct his/her mistake.

How many more blue units than yellow units are in the picture?

Yuki says there are 5 more blue units than yellow units.



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# Lesson 9

### LINEAR MEASUREMENT AND PLACE VALUE

MEASURING LENGTHS IN UNITS OF 10 AND UNITS OF 1 - PART 1



## Lesson 10: Units of 10 and Units of 1

We are going to measure using *units of ten* and *units of one*.

(Hold up a single cube)

If this cube is 1 unit of one, let **connecting them together**.

ake 10 units of one by





This is 10 units of one. Now let's wrap tape around the stick of 10 units to <u>compose</u> 1 unit of ten. This is 1 unit of ten.

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## Lesson 10: Units of 10 and Units of 1

Now let's measure the lengths of all of the objects shown on the worksheet using *units of ten* and *units of one.* 

Start your measurements with the unit of ten and *iterate* as many units of ten as possible before measuring the remaining length by *iterating* units of one.

Write how many of each unit you used and also the total measurement in the space provided.

If you measured something that was 3 units of ten and 2 units of one, what would be the total length of that object?



3 units of ten and 2 units of one is a total length of <u>32</u>.

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Lesson 10 Worksheet: Measuring Lengths in Units of 10 and Units of 1 – Part 1

Name:									
Measure this	Units of 10	Units of 1	Total	Measure this	Units of 10	Units of 1	Total		
Shortest wall in the room				How <b>tall</b> a friend is					
Student's desk				Table					
Teacher's desk				Chair					

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# Lesson 11

### LINEAR MEASUREMENT AND PLACE VALUE

MEASURING LENGTHS IN UNITS OF 10 AND UNITS OF 1 - PART 2



## Lesson 11: Units of 10 and Units of 1 Part 2

Now we will use different units of ten and units of one

to measure objects.



Notice that these units of 10 and units of 1 are smaller

than what we used in the last lesson.

How will this change our measurements if we measured the same objects?

Our measurements will have more units if we use the smaller pieces because it takes more smaller units to measure the same length.

## Lesson 11: Units of 10 and Units of 1 Part 2

Besides using different measuring units than last time, in this lesson you will also need to draw in the units of ten and one you used to measure.

So, instead of getting only 1 unit of ten and 1 unit of one, you now have many of each so you can iterate them for the entire length.

Then, draw a smaller picture of the units to show how many units of ten and units of one you used to make your measurement.

You can trace the edge of the units to help you draw them the correct size.



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Lesson 11 Worksheet: Measuring Lengths in Units of 10 and Units of 1 – Part 2

Name:						
Measure this	Picture of Units					
	ten unit(s) one unit(s)	Length				
	10 1					
	ten unit(s) one unit(s)					
		Length				
	ten unit(s) one unit(s)	Lenath				

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Name:						
Measure this…	Picture of Units					
	ten unit(s) one unit(s)	Length				
	ten unit(s) one unit(s)	Length				
	ten unit(s) one unit(s)	Length				

# Lesson 12

### LINEAR MEASUREMENT AND PLACE VALUE COMPARING LENGTHS IN UNITS OF 10 AND UNITS OF 1



Marsha makes pottery to sell to art collectors around the world.

She recently made a small jar and a large jar.

She needs to fit the jars into boxes and must measure them to make sure they will fit into the boxes.

The small jar is 17 units tall and the large jar is 21 units tall.

Marsha needs to find:

1. How tall the jars are together.

2. How much taller the large jar is than the smaller jar.

How can Marsha solve these two problems?





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17

units

To find the total height of both jars, we can add the two numbers.

17 + 21 = 38

To find the difference between the large and small jar we can either count up:

17 + 4 = 21

Or we can count back:

21 - 4 = 17





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17

We have been measuring in units of ten and units of one.

Now that we know how to compare lengths to find the difference between two measurements, we are going to practice comparing different models using place value.

We have been drawing bar models.

Let's learn how to use the pieces and our models to *compare* lengths of bar models.

### 1. Draw these place value *bar models* for the numbers 23 and 18



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2. Use what you know about units of 10 and units of 1 to find how much *greater* 23 is than 18. This is called the *difference* of 23 and 18.



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*"23 is 5 more units of one than 18. I know this because you have to add 5 units of one to 18 to make it the same as 23."* 



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Draw bar models for the following pairs of numbers.

Use what you know about units of 10 and units of 1 to find the *difference* between each pair.

Describe the *difference* with a partner using your bar models to help you explain your answer.

- a. 21 and 13
- b. 27 and 11

c. 30 and 14

d. 32 and 17

e. 43 and 24

# Lesson 13

### LINEAR MEASUREMENT AND PLACE VALUE EXAMINING ERRORS PART 2

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# Lesson 13: Examining Errors Part 2

Now that we know how to draw bar models, we are going to use them to help explain some incorrect statements that other students have made.

You will want to use the pieces to draw your units of ten and units of one.

It might be helpful to discuss your ideas with a partner as you work.

# Lesson 13: Examining Errors Part 2

For each of the following students' statements, use bar models to show what is incorrect about the students' thinking.

Student Incorrect Statement

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"The Developing Mathematical Thinking Institute (DMTI) is dedicated to enhancing students' learning of mathematics by supporting educators in the implementation of research-based instructional strategies through high-quality professional development, curricular resources and assessments."

For more information contact Dr. Brendefur at jonathan@dmtinstitute.com

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