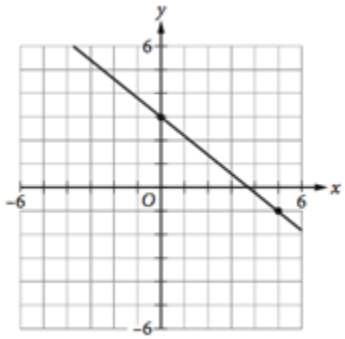
Advanced Algebra and Functions (AAF) ACCUPLACER REVIEW:
All items have solutions in the answer key that follows the problems. Click on a section title to jump to the corresponding answers, and vice-versa.

Problems

I. Linear Equations and Inequalities

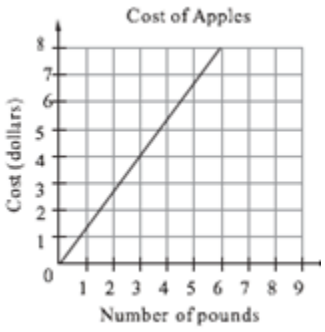
1. Find the slope of a line passing through the points $(1, 2)$ and $(3, 4)$.
2. Find the slope of a line passing through the points $(4, -2)$ and $(-1, 7)$.
3. Write the slope intercept equation of the line that passes through the point $(0, 0)$ and is perpendicular to the line shown below.



4. Write the slope-intercept equation of the line that passes through the point $(1, -2)$ and is parallel to the line $y = \frac{4}{3}x + 11$.
5. Solve the system of equations: $\begin{cases} 3x + 4y = -6 \\ 5x + 3y = 1 \end{cases}$
6. A system of equations in the $xy-$plane has a single solution (x, y).
   If the system is given by the equations $\begin{cases} 2x + y = k \\ x - y = 3 - k \end{cases}$ for some unknown value k, what is the value of the x-coordinate of the solution of the system?
7. Solve: $\frac{3}{5}x + 4 \leq 2$.
8. Solve: $3(x + 3) > 5(x - 1)$.
9. Solve: $2 \leq 3x - 10 \leq 5$.

II. Linear Applications

1. The graph at right shows the cost, in dollars, of apples as a function of the number of pounds of apples purchased at a particular grocery store. The equation $C(p) = \frac{7}{5}p$ defines the cost C, in dollars, for p pounds of pears at the same store. Approximately how much less is the cost per pound of apples than the cost per pound of pears?



2. The total cost in dollars to produce x toasters is given by the function $C(x) = 30 + 1.2x$. Find the cost *per toaster* when 40 toasters are being produced.

3. Last year Julia had already exercised 35 hours prior to May 1. She then exercised 1/2 hour per day on some days during the rest of the year. Write the equation that gives the total number of hours, h, Julia exercised last year if she exercised d days the rest of the year ($0 \leq d \leq 245$).

4. Ethan bought 3 pints of raspberries for \$5 per pint and x pints of blueberries for \$3 per pint. The average cost of all the berries he bought was $\dfrac{15+3x}{3+x}$ dollars per unit. If $y = \dfrac{15+3x}{3+x}$ is graphed in the $xy-$plane, what quantity will be represented by the y-intercept of the graph?

   (a) The total cost, in dollars, of all the raspberries Ethan bought.
   (b) The average cost, in dollars per pint, of all the raspberries Ethan bought.
   (c) The total cost, in dollars, of all the blueberries Ethan bought.
   (d) The average cost, in dollars per pint, of all the blueberries Ethan bought.

5. Sam made \$10 more than twice what Pete earned in one month. If together they earned \$760, how much did Sam earn that month?

6. A woman burns up three times as many calories running as she does when walking the same distance. If she runs 2 miles and walks 5 miles to burn up a total of 770 calories, how many calories does she burn up while running 1 mile?

III. Factoring

1. Expand: $(3x-4)(3x+4)$
2. Expand: $\left(\sqrt{3}x+\sqrt{3}\right)\left(\sqrt{6}x-\sqrt{6}\right)$
3. Which of the following is NOT equivalent to $(3x-12)(x+4)$?

   (a) $3(x^2-8x+16)$ (b) $3(x^2-16)$ (c) $3x^2-48$ (d) $3x(x+4)-12(x+4)$

4. Expand: $(x+5)^2$.
5. Expand: $\left(\dfrac{x}{3}-3y\right)^2$.
6. Expand: $(x+7)(x^2-3x+2)$.
7. Factor completely: $3x^2+6x-24$.
8. Factor completely: $3x^2+25x-18$.
9. Factor completely: $16-4x^2$.
10. Factor completely: $25x^2-4y^2$.
11. Factor completely: $x^2y^2-r^2s^2$.
12. Factor completely: $36x^4+24x^2$.
13. Factor completely: $2x^3+6x^2-8x$.
14. Factor completely: $7x^3y^3+21x^2y^2-10x^3y^2-30x^2y$.

IV. Quadratics

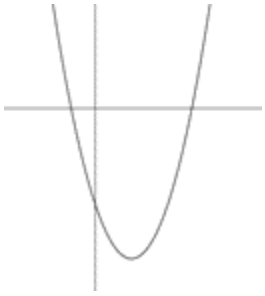
1. Solve: $x^2 + 6x + 8 = 0$
2. Solve: $x^2 + 5x - 9 = 0$
3. Solve: $3x^2 + 8x + 4 = 0$
4. Solve: $(3x + 2)(x - 3) = 7x - 1$
5. For which of the following equations is $x = 6$ the only solution?

   (a) $(6x)^2 = 0$ (b) $(x - 6)^2 = 0$ (c) $(x + 6)^2 = 0$ (d) $(x + 6)(x - 6) = 0$

6. For which of the following equations are $x = 3$ and $x = -3$ both solutions?

   (a) $x^2 - 3x - 9 = 0$ (b) $x^2 + 9 = 0$ (c) $x^2 + 6x - 9 = 0$ (d) $x^2 - 9 = 0$

7. Consider the parabola defined by $f(x) = x^2 - 6x + 4$. Find the vertex.
8. Consider the parabola defined by $f(x) = 3x^2 + x - 1$. Find the vertex.
9. The quadratic function $f(x) = ax^2 + bx + c$ is graphed in the $xy-$plane below, where $y = f(x)$. The equation $ax^2 + bx + c = dx$, where d is a nonzero constant, has how many solutions?



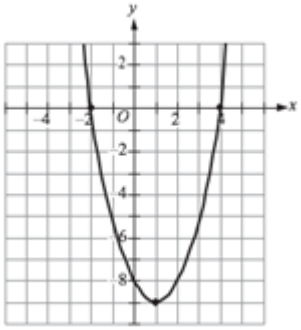
10. The graph of $y = f(x)$ is shown in the $xy-$plane below. Which of the following equations could define $f(x)$?

    (a) $f(x) = x^2 - 2x - 8$

    (b) $f(x) = -x^2 + 2x - 8$

    (c) $f(x) = (x - 2)(x + 4)$

    (d) $f(x) = -(x - 1)^2 - 9$



V. Functions

1. Function g is defined by $g(x) = 3(x + 8)$. What is the value of $g(12)$?

2. Which of the following is the graph of a function $y = f(x)$?

(a) (b) (c) (d)

3. If $f(x) = x^2 + 3x + 1$, what if $f(x + 2)$?

4. For which functions $f(x)$ does $f(a + b) = f(a) + f(b)$ for all real numbers a and b?

   (a) $f(x) = x + 3$ (b) $f(x) = 2x$ (c) $f(x) = x^2$ (d) $f(x) = x^2 + 3$

5. Let $f(x) = 2x + 9$ and $g(x) = 16 - x^2$. Form and simplify the composite function $(f \circ g)(x)$.

VI. Radical and Rational Equations

1. For any unknown x such that $0 < x < 1$, which of the following is the median of the set of numbers $x^3, x^2, x, \frac{1}{x}, \frac{1}{x^2}$?

   (a) x^2 (b) x^3 (c) $\frac{1}{x}$ (d) x

2. Simplify the expression $\dfrac{8x^2 + 4x}{2x}$.

3. Simplify the expression $\dfrac{x}{y} + \dfrac{x - y}{y} + \dfrac{2x}{3y}$.

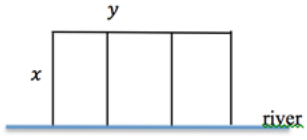
4. Simplify the expression $\dfrac{x - y}{\sqrt{x} - \sqrt{y}}$.

5. Simplify the expression $\dfrac{1 + \dfrac{x}{y}}{1 - \dfrac{x}{y}}$.

6. Simplify the expression $\dfrac{x - \dfrac{1}{y}}{\dfrac{1}{x} - y}$.

7. Simplify the expression $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{\dfrac{x + y}{y}}$.

8. A rectangular plot of farmland is to be enclosed with fencing on three sides, with a river bordering the fourth side. Additional fencing is to be placed as shown in the figure at right. If the area of this plot is to be 4,000 square feet, how much fencing is needed, in terms of the variable y?



9. What is the domain of the function $f(x) = \dfrac{2x-5}{x^2-9}$?

10. If $f(x) = \dfrac{x}{1-x}$ and $g(x) = \dfrac{1}{x}$, what is the domain of the function $f+g$?

11. Simplify the expression $\sqrt{x^3+x^2} - \sqrt{4+4x}$.

12. Find all real solutions to $\sqrt{5x+1} + 9 = 3$.

13. Find all real solutions to $\sqrt[3]{3x+2} + 4 = 6$.

14. Find all real solutions to $\sqrt[4]{x^2+7} = 2$.

15. The velocity of a free-falling object is represented by the equation $V = \sqrt{2gh}$, where V is the velocity, g is the rate of acceleration due to gravity (32 feet per second squared), and h is the height in feet. What is the height from which an object is dropped if the object hits the ground with a velocity of 75 feet per second?

16. Solve: $\dfrac{5}{x+2} = \dfrac{x}{2x-3}$.

17. Solve: $\dfrac{1}{y-1} + \dfrac{2}{y+1} = 0$.

18. Solve: $\dfrac{-1}{x^2-3x} = \dfrac{1}{x} + \dfrac{x}{x-3}$.

VII. Polynomial Equations

1. Find the range of $y = -2x^4 + 7$.

2. If $f(x) = x^3 - 4x^2 + 2x + 1$, then $f(-x) =$?

3. In the function $f(x) = a(x+2)(x-3)^b$, a and b are both integer constants and b is positive. If the end behavior of the graph of $y = f(x)$ is **positive** for both very large negative values of x and very large positive values of x, what is true about a and b?

   (a) a is negative, and b is even.
   (b) a is positive, and b is even.
   (c) a is negative, and b is odd.
   (d) a is positive, and b is odd.

4. Find a polynomial function of degree 3 whose zeros are -3, 2, and 5.

5. Which of the following is a factor of $f(x) = 2x^3 - x^2 + 2x - 3$?

   (a) $x-1$ (b) $x+3$ (c) $x+1$ (d) $x-3$

6. Which of the following is a factor of $f(x) = x^3 - 5x^2 + 8x - 16$?

   (a) $x+4$ (b) $x+2$ (c) $x-4$ (d) $x-2$

7. Solve the inequality: $x^2 - 2 < \dfrac{7}{2}x$.

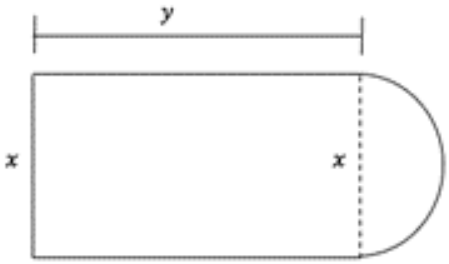
8. Solve the inequality: $6x^2 + 5x \geq 4$.

VIII. Exponential and Logarithmic Equations

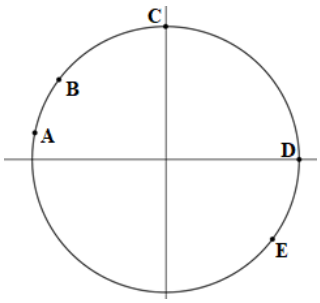
1. Express $\log_5 25 = 2$ in exponential form.
2. Write the following in logarithmic form: $2^{5x} = 7$.
3. A biologist puts an initial population of 500 bacteria into a growth plate. The population is expected to double every 4 hours. Write the equation that will give the expected number of bacteria, n, after x days. (24 hours = 1 day)
4. The population of Salt Lake City, Utah, in 2013 was approximately 191,180. If the population increases at an annual rate of 1.1%, what will be the population of Salt Lake City in 2020?
5. Suppose that \$1000 is invested at an annual rate of 3%, compounded annually. Find the total amount in the account after 2 years.
6. If $\log_4(x-2) = 3$, then what is the value of x?
7. Solve: $\log_2 x = 4$.
8. Use the properties of logarithms to expand as much as possible: $\log_4 \dfrac{3}{y^5}$.
9. Use the properties of logarithms to write the following as a single logarithm: $3\log x + 2\log(x+1)$.
10. How long will it take for \$850 to be worth \$1,000 if it is invested at 12% interest compounded annually?

IX. Geometry Concepts

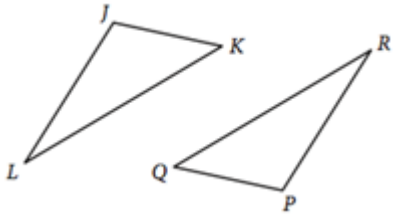
1. The surface area of a right rectangular prism can be found by finding the sum of the area of each of the faces of the prism. What is the surface area of a right rectangular prism with length 4 centimeters (cm), width 9 cm, and height 3 cm? (Area of a rectangle is equal to length times width.)
2. The radius and height of a certain right cylinder are known to be equal. If the volume of the cylinder is 27π cubic inches, find the height of the cylinder.
3. The figure below consists of a semicircle and a rectangle. If the perimeter of the figure is 16, what is y in terms of x?



4. In the figure below, which of the lettered points could be the intersection of the circle $x^2 + y^2 = 25$ and the line $y = 3$?



5. Triangle JKL and triangle PQR are shown below. If $\angle J$ is congruent to $\angle P$ which of the following must be true to prove that triangles JKL and PQR are congruent?



(a) $\angle L \cong \angle R$ and $JL = PR$

(b) $KL = QR$ and $PR = JL$

(c) $JK = PQ$ and $KL = QR$

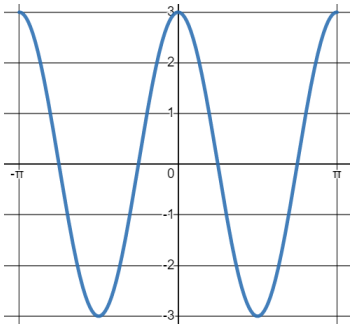
(d) $\angle K \cong \angle Q$ and $\angle L \cong \angle R$

X. Trigonometry

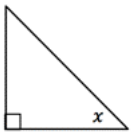
1. In triangle ABC, and C is a right angle. If $\cos A = \frac{5}{8}$, what is the value of $\cos B$?

2. What is $\frac{13}{18}\pi$ in degrees?

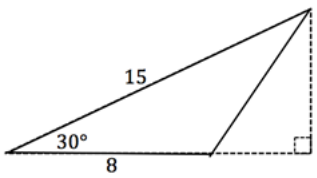
3. The graph shown below shows a portion of the graph of what function?



4. Consider the triangle given to the right.
If $\cot(x^\circ) = 1$, then what is the measure of angle x?



5. Find the area of the obtuse triangle to the right.



Answers

I. Linear Equations and Inequalities

1. $m = 1$
2. $m = -\frac{9}{5}$
3. $y = \frac{5}{4}x$
4. $y = \frac{4}{3}x - \frac{10}{3}$
5. $x = 2$, $y = -3$
6. $x = 1$
7. $x \leq -\frac{10}{3}$
8. $x < 7$
9. $4 \leq x \leq 5$

II. Linear Applications

1. Approximately \$0.07 per pound
2. \$1.95 per toaster
3. $h = 35 + \frac{1}{2}d$
4. (b) The average cost, in dollars per pint, of all the raspberries Ethan bought.
5. \$510
6. 210 calories

III. Factoring

1. $9x^2 - 16$
2. $\sqrt{18}x^2 - \sqrt{18}$
3. (a) $3(x^2 - 8x + 16)$
4. $x^2 + 10x + 25$
5. $\frac{x^2}{9} - 2xy + 9y^2$
6. $x^3 + 4x^2 - 19x + 14$
7. $3(x + 4)(x - 2)$
8. $(3x - 2)(x + 9)$
9. $(4 - 2x)(4 + 2x)$

III. Factoring (cont.)

1. $(5x + 2y)(5x - 2y)$
2. $(xy - rs)(xy + rs)$
3. $12x^2(3x^2 + 2)$
4. $2x(x + 4)(x - 1)$
5. $(xy + 3)(7x^2y^2 - 10x^2y)$

IV. Quadratics

1. $x = -2$, $x = -4$
2. $x = 2$, $x = -7$
3. $x = -2$, $x = -\frac{2}{3}$
4. $x = 5$, $x = -\frac{1}{3}$
5. (b) $(x - 6)^2 = 0$
6. (d) $x^2 - 9 = 0$
7. $(3, -5)$
8. $\left(-\frac{1}{6}, -\frac{13}{12}\right)$
9. 2 solutions
10. (a) $f(x) = x^2 - 2x - 8$

V. Functions

1. $g(12) = 60$
2. (c)
3. $f(x + 2) = x^2 + 7x + 11$
4. (b) $f(x) = 2x$
5. $(f \circ g)(x) = 41 - 2x^2$

VI. Radical and Rational Equations

1. (d) x
2. $4x + 2$
3. $\frac{8x}{3y} - 1$
4. $\sqrt{x} + \sqrt{y}$

VI. Radical and Rational Equations (cont.)

1. $\dfrac{y+x}{y-x}$
2. $-\dfrac{x}{y}$
3. $\dfrac{x-y}{x}$
4. $y + \dfrac{16000}{y}$
5. $x \neq 3$, $x \neq -3$
6. $x \neq 0$, $x \neq 1$
7. $(x-2)\sqrt{x+1}$
8. There is no real solution.
9. $x = 2$
10. $x = 3$, $x = -3$
11. height = 87.89 feet
12. $x = 3$, $x = 5$
13. $y = \dfrac{1}{3}$
14. $x = 1$, $x = -2$

VII. Polynomial Equations

1. Range: $(-\infty, 7]$
2. $f(-x) = -x^3 - 4x^2 - 2x + 1$
3. (d) a is positive, and b is odd.
4. $f(x) = (x+3)(x-2)(x-5)$
5. (a) $x - 1$
6. (c) $x - 4$
7. $-\dfrac{1}{2} < x < 4$
8. $x \leq -\dfrac{4}{3}$ or $x \geq \dfrac{1}{2}$

VIII. Exponential and Logarithmic Equations

1. $5^2 = 25$
2. $\log_2(7) = 5x$
3. $n = 500(2)^{6x}$
4. 206,396
5. \$1060.90
6. $x = 66$
7. $x = 16$
8. $\log_4 3 - 5\log_4 y$
9. $\log\left(x^3(x+1)^2\right)$
10. Approximately 1.43 years

IX. Geometry Concepts

1. 150 cm
2. height = 3 inches
3. $y = 8 - \dfrac{x}{2} - \dfrac{\pi x}{2}$
4. B
5. (a) $\angle L \cong \angle R$ and $JL = PR$

X. Trigonometry

1. $\cos B = \dfrac{\sqrt{39}}{8}$
2. 130°
3. $y = 3\cos(2x)$
4. $x = 45^\circ$
5. Area = 30